

Practice Final Exam Solutions

Problem 1

There are 3 subgame perfect equilibria: $(A, \frac{1}{2}L + \frac{1}{2}R, \frac{1}{2}a + \frac{1}{2}b)$, (B, L, b) and (B, R, a) .

Problem 2

- (a) The socially optimal level of investment maximizes $v(x) - c(x) - x$. It is determined by the first-order condition $v'(x) - c'(x) = 1$.
- (b) If the buyer makes an offer of p , the seller's payoff is $p - c(x) - x$ if he accepts the offer and $-x$ if he rejects the offer. The buyer will make the smallest offer that the seller will accept, i.e. $p = c(x)$. Then the seller's payoff in the second stage is $-x$, which is maximized when the seller makes investment $x = 0$.
- (c) If the seller makes an offer of p , the buyer's payoff is $v(x) - p$ if he accepts the offer and 0 if he rejects. The seller will offer the largest price that the seller will accept, i.e. $p = v(x)$. The seller's payoff in the second stage is $v(x) - c(x) - x$, which is maximized when he chooses the socially optimal investment level.

Problem 3

- A. No, there is no PBE where the normal type of player 1 always takes the money in node 1. Suppose the normal type of player 1 always took the money in node 1. Then if player 1 passes, player 2 would become instantaneously convinced that player 1 is behavioral. Given that, it is best for player 2 to pass in every node. Then the normal type of player 1 would have a profitable deviation of passing in node 1.
- B. If player 2 takes the money in node 4, he gets 4 for sure. If he passes, his expected payoff is $5p_4$. Player 2 will pass if $p_4 > 4/5$, indifferent if $p_4 = p_4^* = 4/5$, and take the money if $p_4 < 4/5$.
- C. The normal type cannot always take the money in node 3. Otherwise passing would convince player 2 that player 1 is behavioral, causing player 2 to pass in node 4, which creates a profitable deviation. The normal type cannot always pass, because then p_4 would be equal to p_3 , and player 2 will take the money in node 4. Therefore, the normal type of player 1 must mix. To make player 1 indifferent, player 2 must mix to give player 1 expected payoff of 3 if player 1 passes. (Player 2 must pass with probability $3/5$ in node 4.) Because player 2 mixes, from part B we know that $p_4 = p_4^*$. Then player 1 will pass with *total* probability q such that $p_3/q = p_4^*$.
- D. If player 2 takes the money, he gets a sure payoff of 2. If player 2 passes, player 1 will pass with probability $q = p_3/p_4^*$, so player 2's expected payoff is $4q$. (Note that player 2's payoff if he reaches node 4 is 4, because in that node he is indifferent between taking the money and passing). We conclude that in node 2

- player 2 passes if $2 < (4q) \Leftrightarrow p_2 > p_4^*/2 = 2/5$, is indifferent if $p_2 = 2/5$ and takes the money if $p_2 < 2/5$.
- E. If $p = 1/2$, player 1 will pass in node 1 because he anticipates player 2 to pass in node 2.

Problem 4.

Consider an education signaling model with three types of workers, who have productivities $x_L < x_M < x_H$. Suppose that education does not add to productivity and has per-unit costs of $c_L > c_M > c_H$ to the three types. Each worker knows his type, but the market has initial belief (p_L, p_M, p_H) with $p_L + p_M + p_H = 1$.

- A. The equilibrium wage in the pooling equilibrium is $w_P = p_L x_L + p_M x_M + p_H x_H$.
To determine the range of education levels for which a pooling equilibrium exists, the main condition to check is that the low type does not want to deviate to the efficient education level 0, i.e. $w_P - c_{Le} \geq x_L$. A pooling equilibrium exists for all education levels in the range $[0, (w_P - x_L)/c_L]$. The pooling equilibrium fails the intuitive criterion. Let e' be the education level defined by $w_P - c_{He} = x_H - c_{He}'$. It is easy to check that only type H could possibly benefit (relative to his equilibrium payoff) by deviating to $e' - \varepsilon$. The intuitive criterion implies that the market must assign probability 1 to type H if it sees education level e' , and offer the wage of x_H . Given those beliefs, type H has a profitable deviation to e' .
- B. The least-cost separating equilibrium, is defined by the following conditions

$$\begin{aligned} e_L &= 0 && \text{(type L chooses the efficient level of education)} \\ x_M - c_{Le_M} &= x_L && \text{(type M signals just enough to separate from type L)} \\ x_H - c_{Me_H} &= x_M - c_{Me_M} && \text{(type H signals just enough to separate from M)} \end{aligned}$$

- This equilibrium satisfies the intuitive criterion. Let the market believe that the worker is type L if $e \in (0, e_M)$. These beliefs satisfy the intuitive criterion: type L would deviate to e if inferred to be type M. Let the market believe that the worker is type M if $e \in (e_M, e_H)$. Again, these beliefs satisfy the intuitive criterion: type M would deviate into this range if inferred to be type H. For $e > e_H$ beliefs could be arbitrary: nobody would deviate to e no matter what beliefs are.
- C. There are other separating equilibria that satisfy the intuitive criterion. For example, let (e_L', e_M', e_H') be defined by

$$\begin{aligned} e_L' &= 0, e_M' = e_M + \varepsilon \\ x_H - c_{Me_H'} &= x_M - c_{Me_M'} \end{aligned}$$

Let the market believe that the worker is type L if $e \in (0, e_M')$. These beliefs satisfy the intuitive criterion: type L would deviate to e if inferred to be type H. Let the market believe that the worker is type M if $e \in (e_M', e_H')$. Again, these beliefs satisfy the intuitive criterion: type M would deviate into this range if inferred to be type H. For $e > e_H'$ beliefs could be arbitrary: nobody would deviate to e no matter what beliefs are.