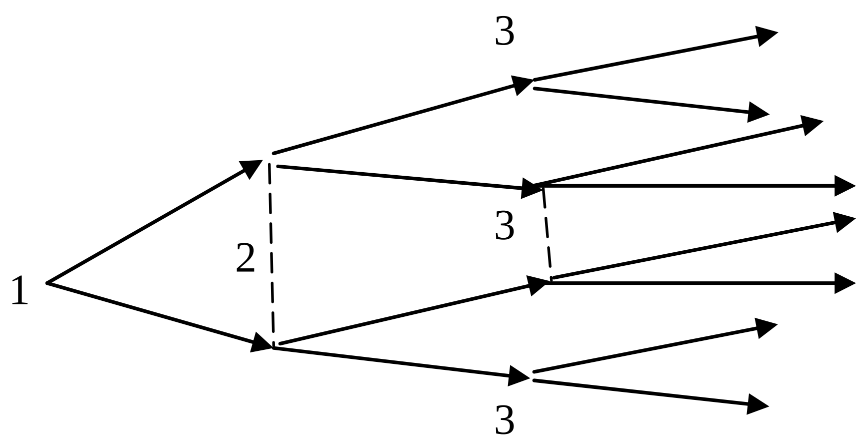
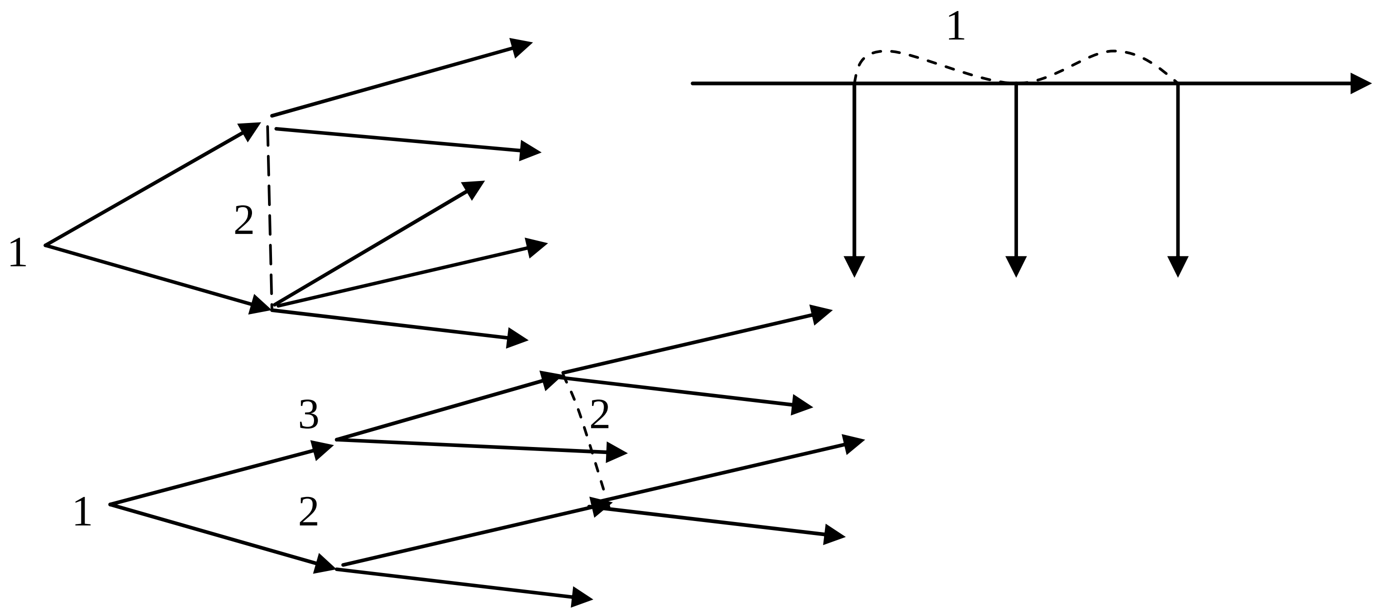
Games with Incomplete Information

A *strategy* of a player in an extensive form game is a complete plan of actions, i.e. it specifies an action in every single information set where that player moves. Formally, it is a function that assigns an action to each information set.



Problematic Information Sets



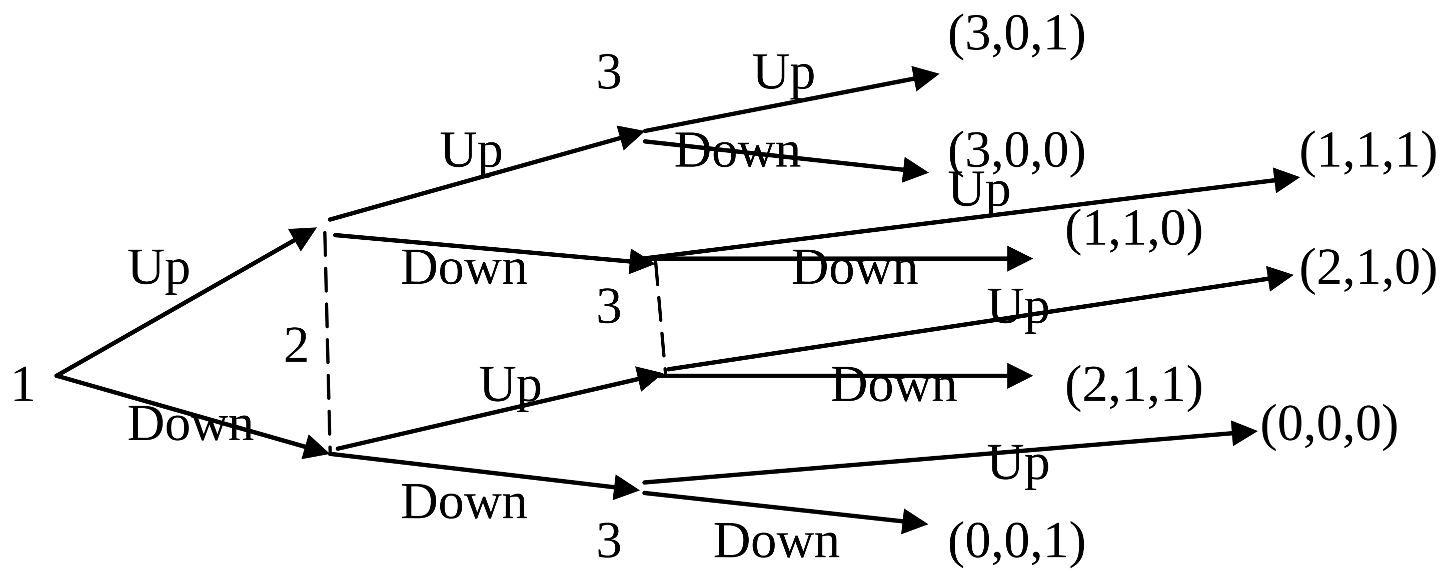
A *subgame* is a subset of the game such that

- it begins with an information set that has one node
- it contains all successor nodes and no other nodes
- it contains only whole information sets

3 times repeated PD: how many subgames?

A *subgame perfect equilibrium* (SPE) is a combination of strategies, one for each player such that in every *subgame* the strategy of each player is a best response to the combination of his opponent's strategies.

Questions about the following game:

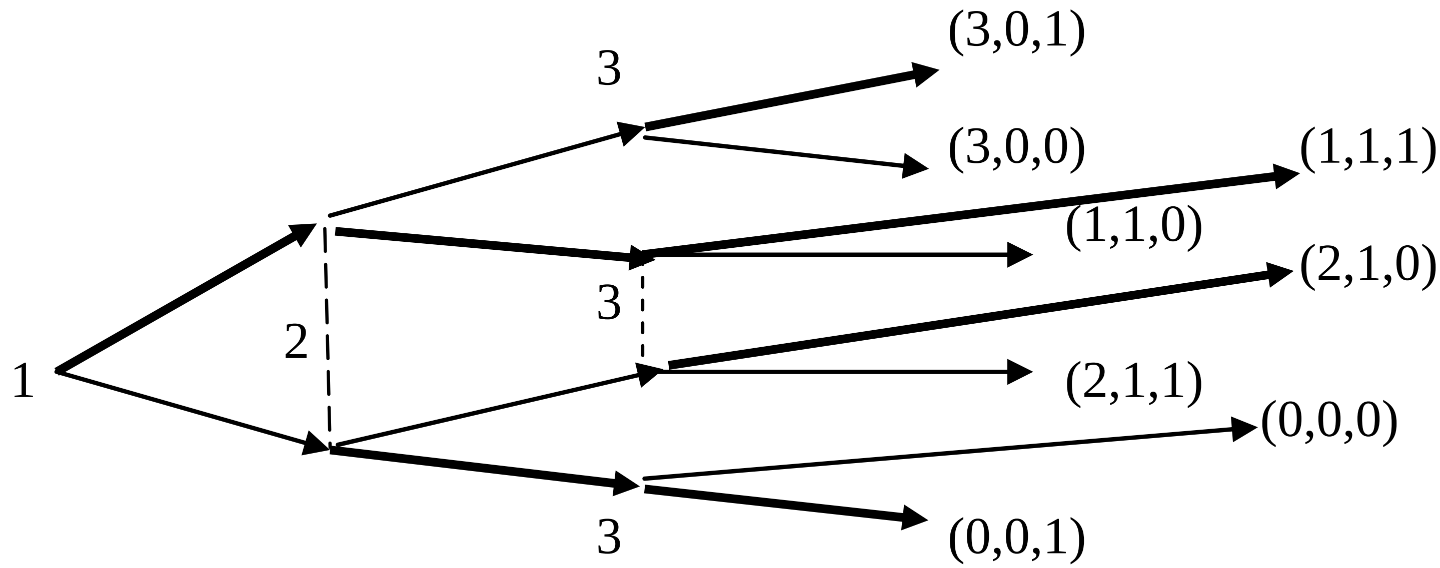


How many subgames?

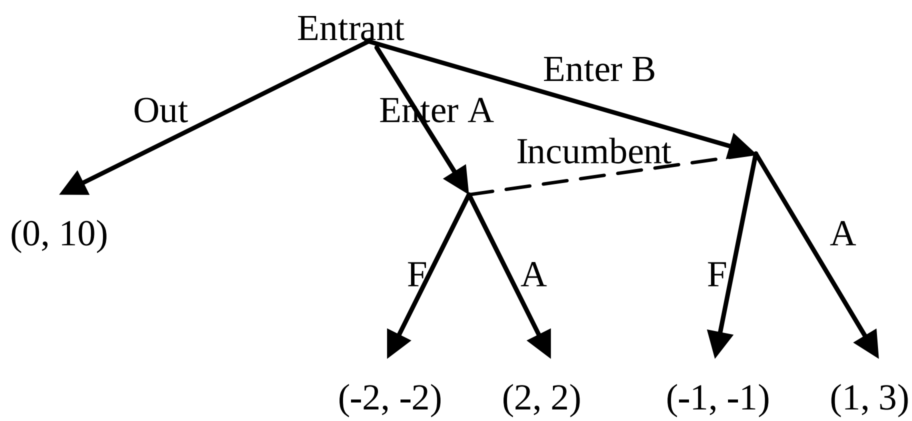
How many strategies does each player have?

Find Subgame Perfect equilibria.

Beliefs



Beliefs



Definition: A *system of beliefs* specifies a probability distribution over the nodes in each information set.

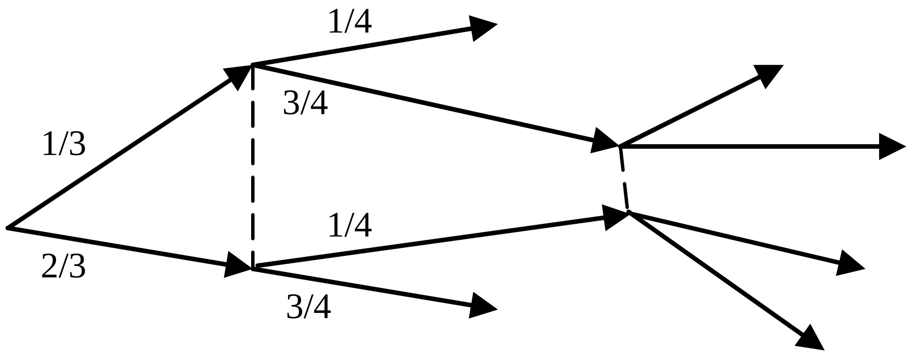
Question: are there any other SPE in the game above?

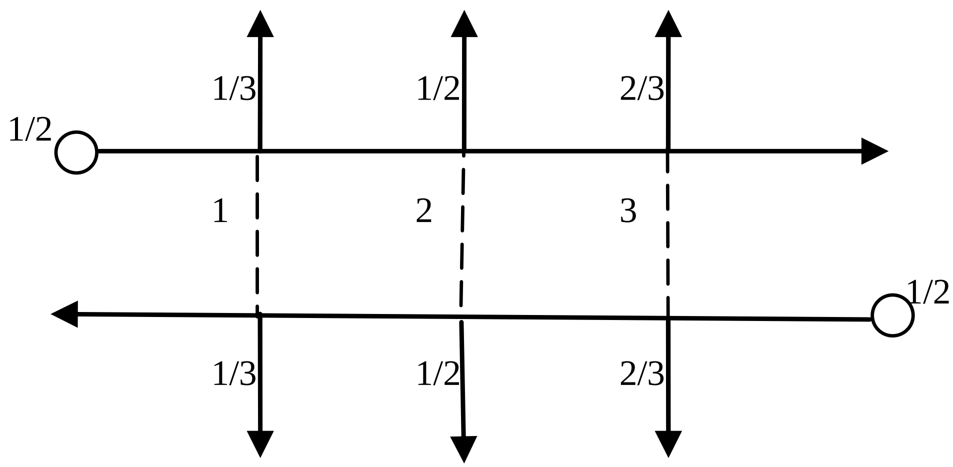
Perfect Bayesian Equilibrium

Definition: A combination of strategies is a *perfect Bayesian equilibrium (PBE)* if

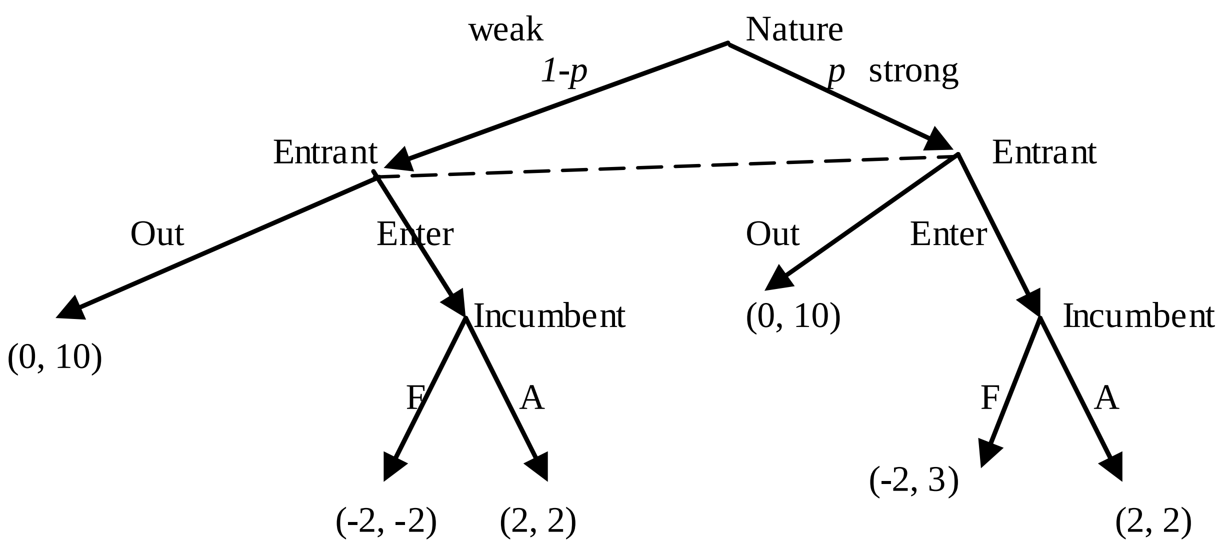
- Beliefs in each information set are formed by Bayes rule from strategies
- The action of each player in each information set is optimal given his belief, and the strategies of all players in the future

How Bayes Rule Works



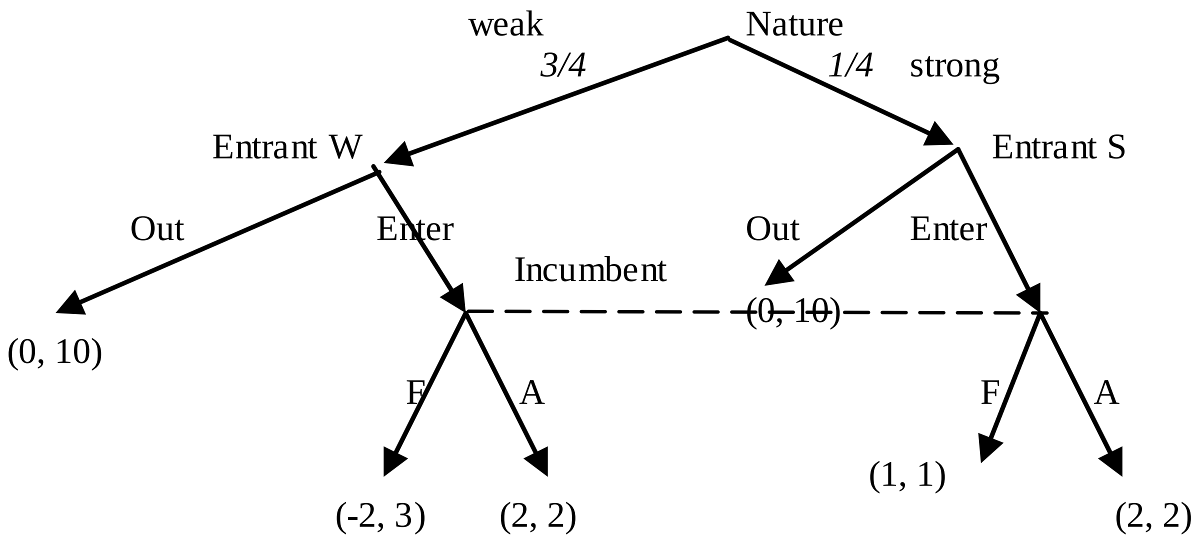


Example

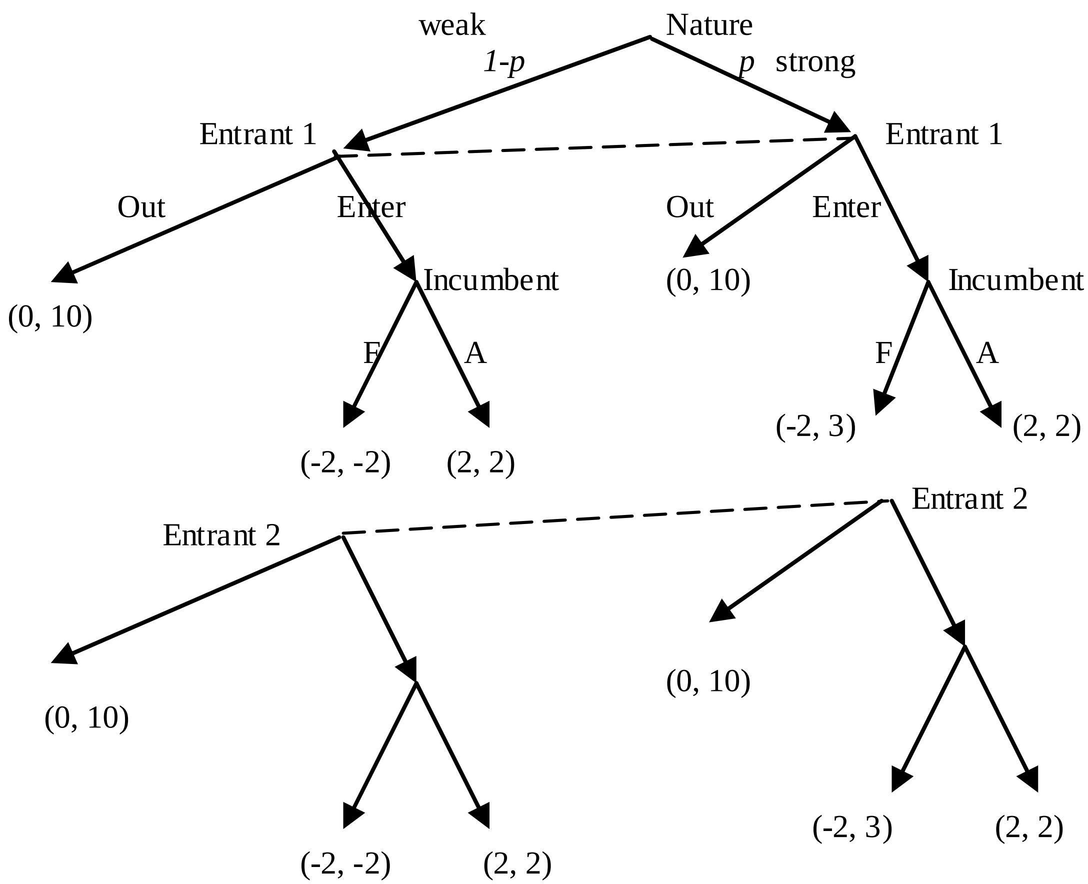


For what values of *p* is there a PBE where the entrant enters?

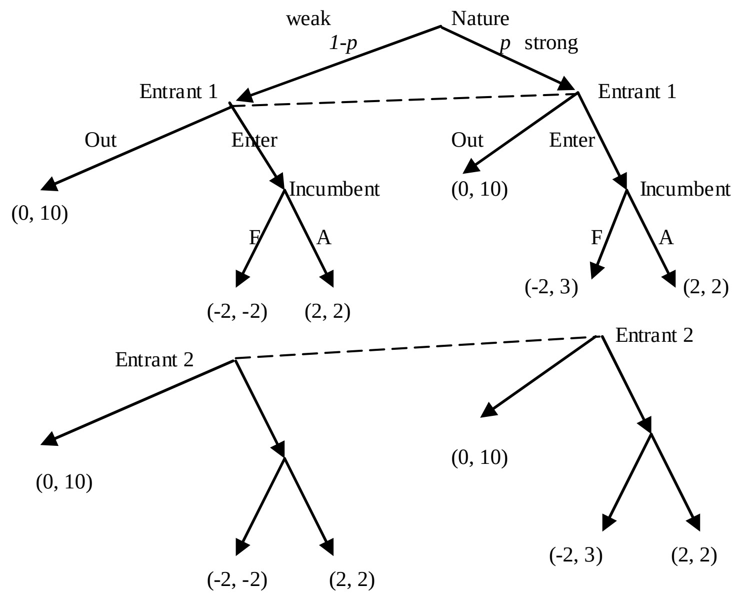
Example



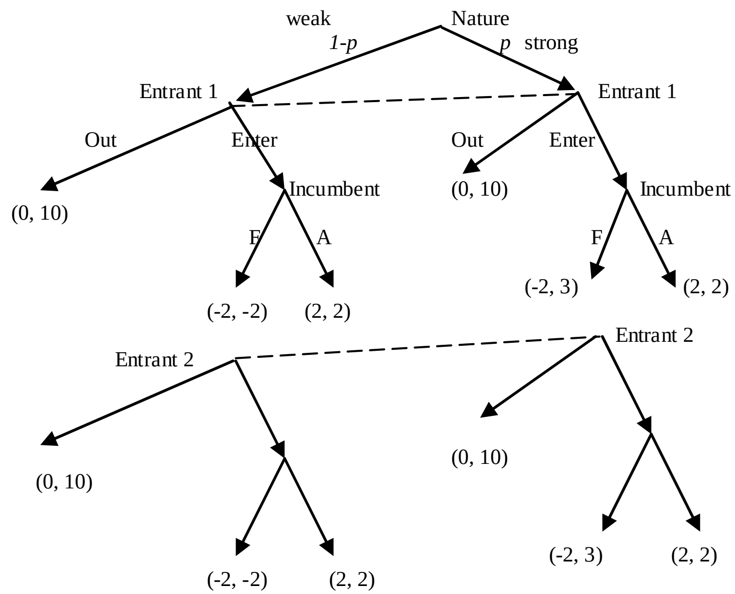
Exercise: Find a PBE.



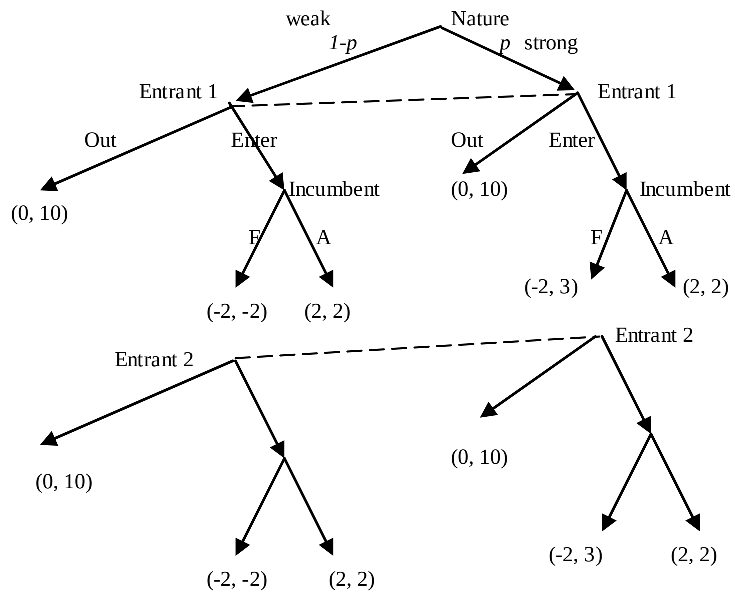
Repeated twice: what happens in PBE if $p = 3/8$?



- If the incumbent is strong, he will always fight.
- Weak incumbent: will he fight in the first period?
- If not, then Entrant 2 concludes *strong* if he sees *fight*.
- So Entrant 2 will not enter in period 2.
- Given that, if *weak* incumbent fights in the first period, he gets -2+10. If he accommodates, he gets 2+2, which is less.
- No PBE where weak incumbent accommodates in period 1.

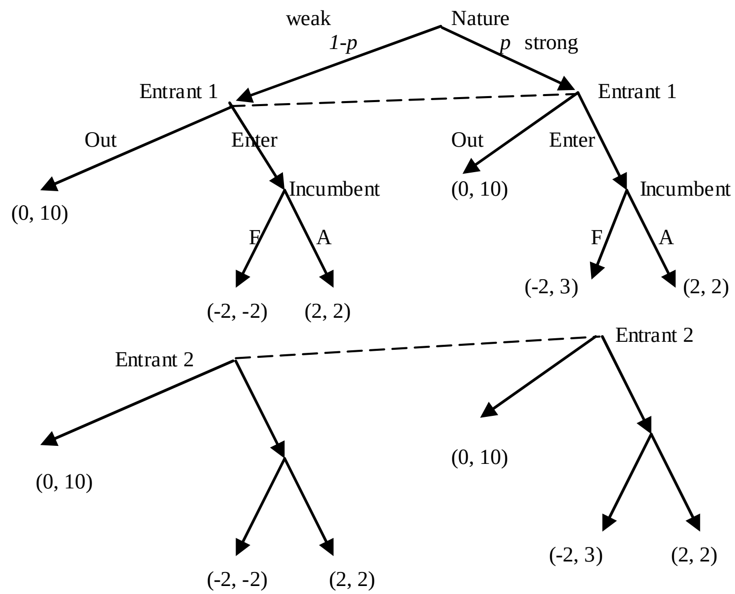


- If the weak incumbent fights in period 1, then both types fight.
- If the Entrant 2 sees fighting in period 1, he does not update beliefs: i.e. believes incumbent is *strong* with prob. *p = 3/8*.
- So Entrant 2 will *enter* in period 2.
- Given that, if *weak* incumbent fights in the first period, he gets -2+2. If he accommodates in period 1, he gets 2+2, which is more.
- No PBE where weak incumbent fights in period 1.



- weak incumbent must mix in period 1 (*qA + (1-q)F)*
- Entrant 2 will use Bayes rule to update beliefs
  - if he sees A in period 1, he believes *weak* for sure
  - if he sees F in period 1, be believes *strong* w/ prob

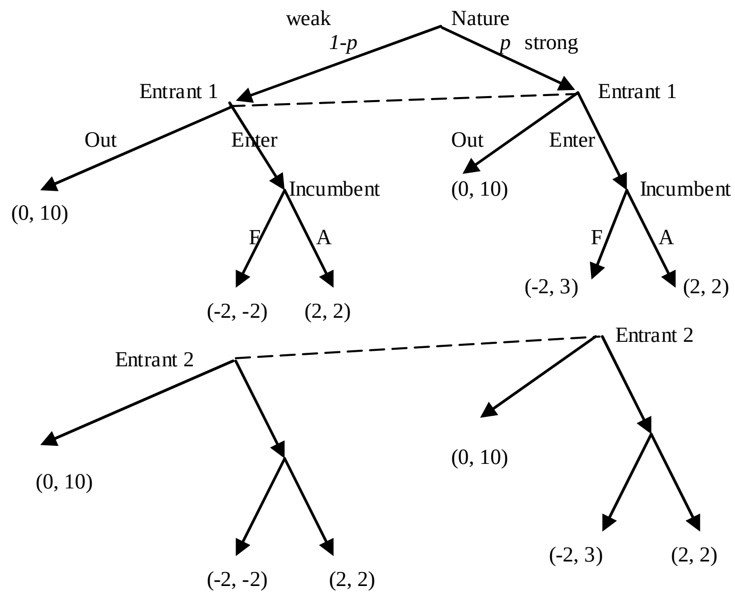
$$p' = \frac{p}{p + (1-p)q}$$



Entrant 2 will enter if $p' = \dfrac{p}{p+(1-p)q} < 1/2$

not enter if $p' = \dfrac{p}{p+(1-p)q} > 1/2$

indifferent if $p' = \dfrac{p}{p+(1-p)q} = 1/2$



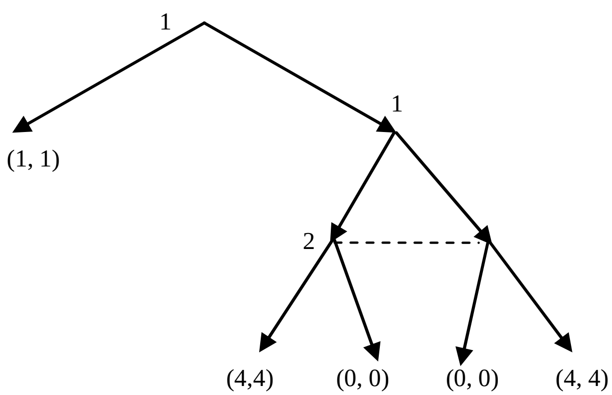
Entrant 2 must mix to cause the weak Incumbent to be indifferent between fighting or not in period 1, so we must have $p' = \frac{3/8}{3/8 + 5/8q} = 1/2 \Rightarrow q = 3/5$.

Exercise: Find the probabilities with which the Entrant 2 must mix (this will give us the *unique* PBE).

Some Formal Theory

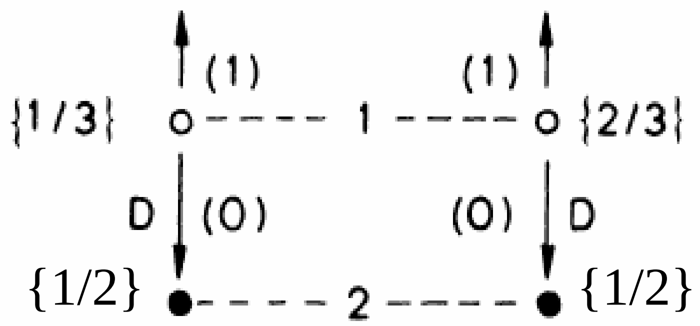
Weak perfect Bayesian equilibrium (wPBE) (See MWG):

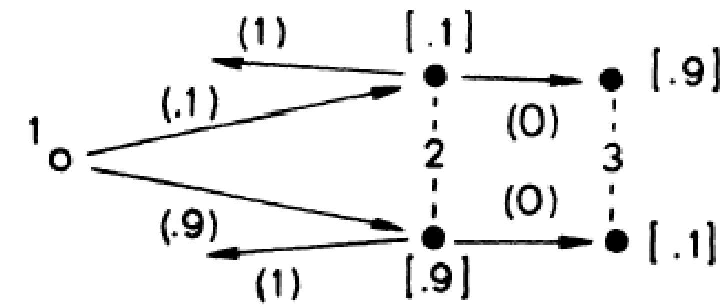
- Beliefs are formed by Bayes rule when possible
- When not possible, beliefs could be anything
- Actions are optimal at information sets given beliefs

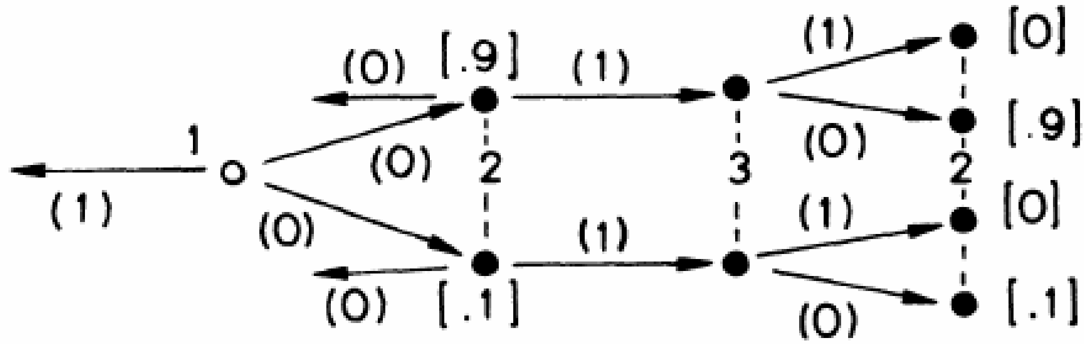


PBE = wPBE in every subgame

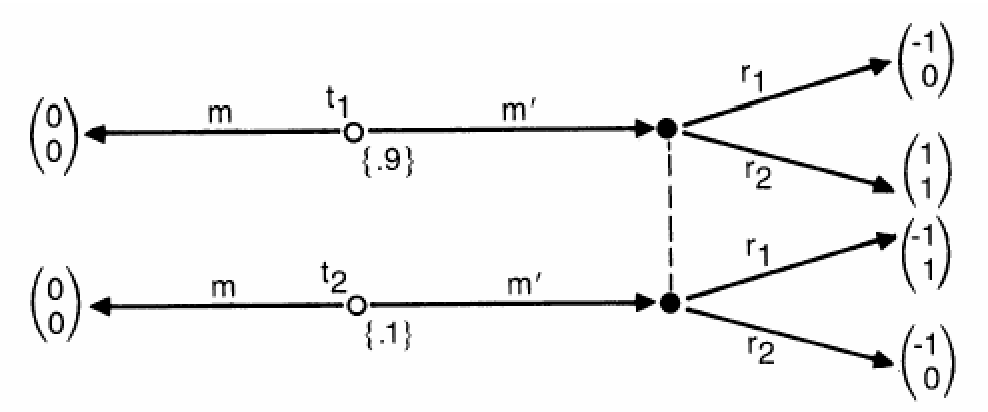
Sequential Equilibrium = PBE + restrictions on beliefs in zero-prob. info sets (defined by game tree, *not* payoffs). Three examples possible in PBE, not in sequential:







Sequential Equilibrium with "unreasonable" beliefs:



(such beliefs are ruled out by the *intuitive criterion*, later in the course)