Bertrand competition

- worker with productivity $x>0$
- two or more employers simultaneously name wages $w_1, w_2, \dots$
- the worker chooses which offer to accept
- if offer firm i is accepted, the payoffs are $(w_i, x-w_i)$

What is a Nash equilibrium?

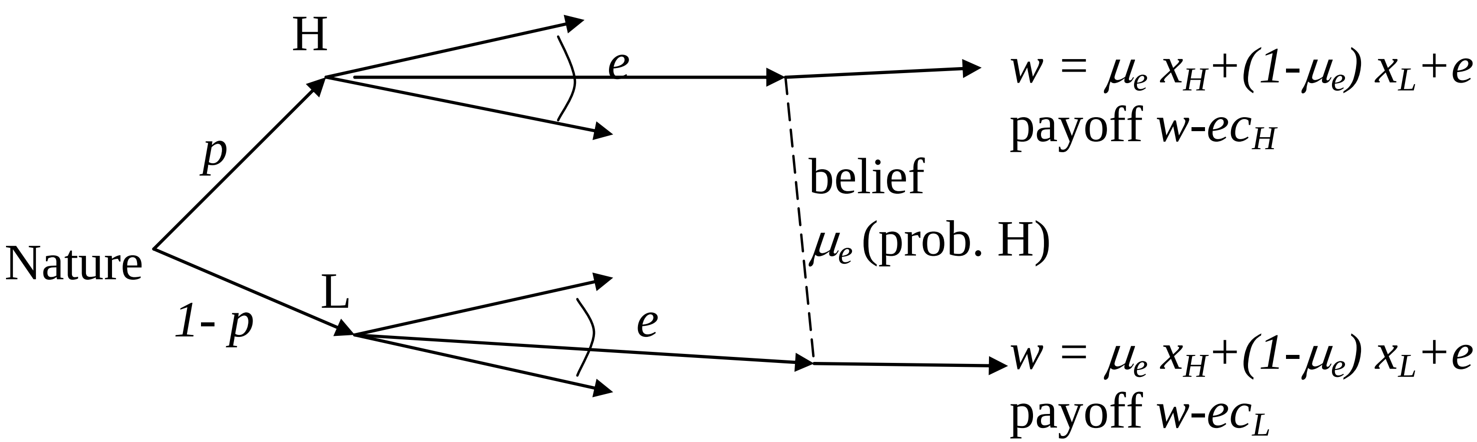
Unknown Productivity

- worker's productivity is x_H with probability p and $x_L < x_H$ with probability $(1-p)$
- two or more employers (who are risk neutral) simultaneously name wages w_1, w_2, …
- the worker chooses which offer to accept
- if offer firm i is accepted, the payoffs are (w_i, x_L-w_i) or (w_i, x_H-w_i) depending on the worker's type

What is a Nash equilibrium?

Signaling

- suppose worker can choose education level $e \in [0, \infty)$, observable to employers (increases productivity by e)
- neither type of worker enjoys education: unit of education costs c_L for worker L and $c_H \in (1, c_L)$ for H
- characterize PBE



2 types of equilibria: pooling & separating

- pooling (both types choose the same level of e^*)
  - need to specify e^*
  - beliefs $\mu_{e*} = p$ (probability of the high type)
  - need to specify beliefs μ_e for $e \neq e^*$
  - check that each type chooses e^* for those beliefs

Payoff for type L: $px_H+(1-p)x_L +e^* - c_L e^*$

If type L deviates to $e=0$, his payoff is at least: x_L

Need: $px_H+(1-p)x_L +e^*-c_L e^* \geq x_L \Leftrightarrow e^* \leq \dfrac{p(x_H - x_L)}{c_L - 1}$

Therefore: no equilibrium with $e^* > \dfrac{p(x_H - x_L)}{c_L - 1}$

Claim: There is a PBE for all $e^* \leq \frac{p(x_H - x_L)}{c_L - 1}$

Proof. Let $\mu_e = 0$ for all $e \neq e^*$. Then equilibrium wage is $x_L + e$ for all $e \neq e^*$. To show that this gives us a pooling PBE, we need to show that neither type of worker would want to deviate from choosing $e = e^*$. Note that $x_L + e - ce$ is maximized when $e=0$, so the best deviation is 0. Since

$$px_H + (1-p)x_L + e^* - c_L e^* > x_L \qquad (*)$$

(prev. slide) type L would not want to deviate. Also, (*) implies that

$$px_H + (1-p)x_L + e^* - c_H e^* > x_L$$

Remarks:

- If $\mu_e = 0$ are off-equilibrium path beliefs, then any $e^* \leq \frac{p(x_H - x_L)}{c_L - 1}$ gives a pooling PBE.
- Off-equilibrium path beliefs are not determined by Bayes rule.
- For any $e^* \leq \frac{p(x_H - x_L)}{c_L - 1}$ there may be other off-equilibrium path beliefs that also give a PBE.

2 types of equilibria: pooling & separating

- separating (both types choose different levels of e)
  - need to specify e_L and e_H
  - beliefs $\mu_{eL} = 0$ and $\mu_{eH} = 1$
  - need to specify beliefs μ_e for $e \neq e_L, e_H$
  - check that each type chooses his education level given beliefs

Claim: $e_L = 0$.

Proof. Type L gets wage x_L if he chooses education level e_L. Then $e_L > 0$ would give utility $x_L + e - c_L e$. By deviating to 0 type L would get at least x_L.

- We must have $0 = e_L < e_H$

Two main things to check:

- Type L would not want to deviate to e_H

$$x_H + e_H - c_L e_H \leq x_L \Leftrightarrow e_H \geq \frac{(x_H - x_L)}{c_L - 1}$$

- Type H would not want to deviate to 0

$$x_H + e_H - c_H e_H \geq x_L \Leftrightarrow e_H \leq \frac{(x_H - x_L)}{c_H - 1}$$

Claim: There is a PBE for all $e_H \in \left[\frac{(x_H - x_L)}{c_L - 1}, \frac{(x_H - x_L)}{c_H - 1} \right]$

Proof. Set $\mu_e = 0$ for all $e \neq e_H$ to make this work.

We had to specify many off-equilibrium beliefs. Are all those beliefs reasonable?

Intuitive criterion:

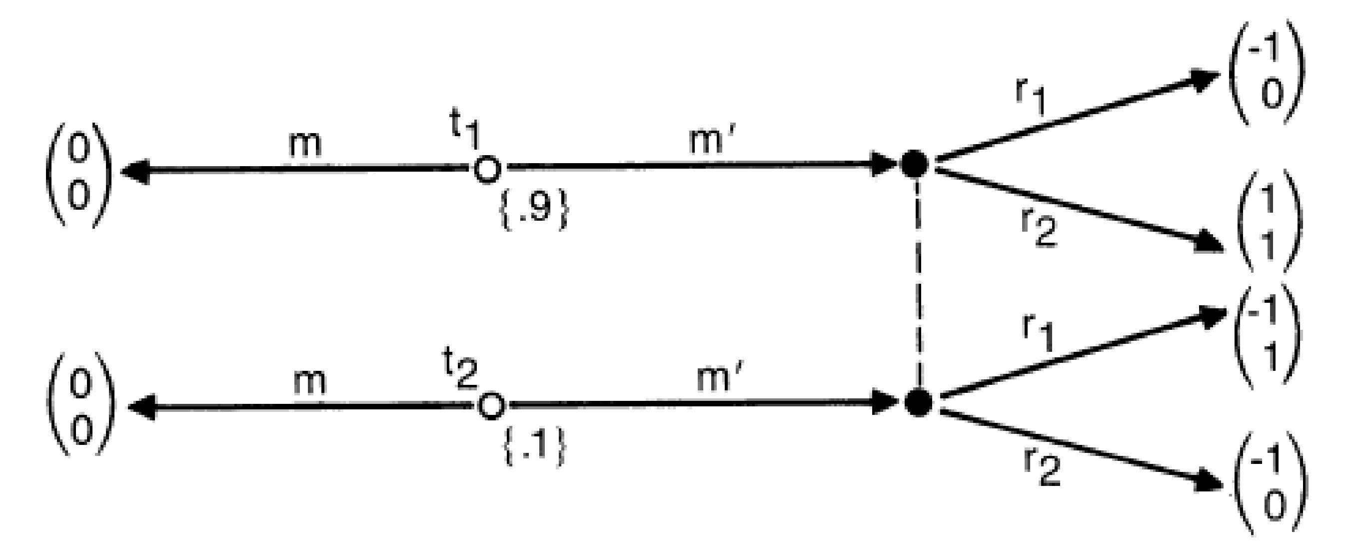
Consider an off-equilibrium information set. Suppose for some type the following statement applies:

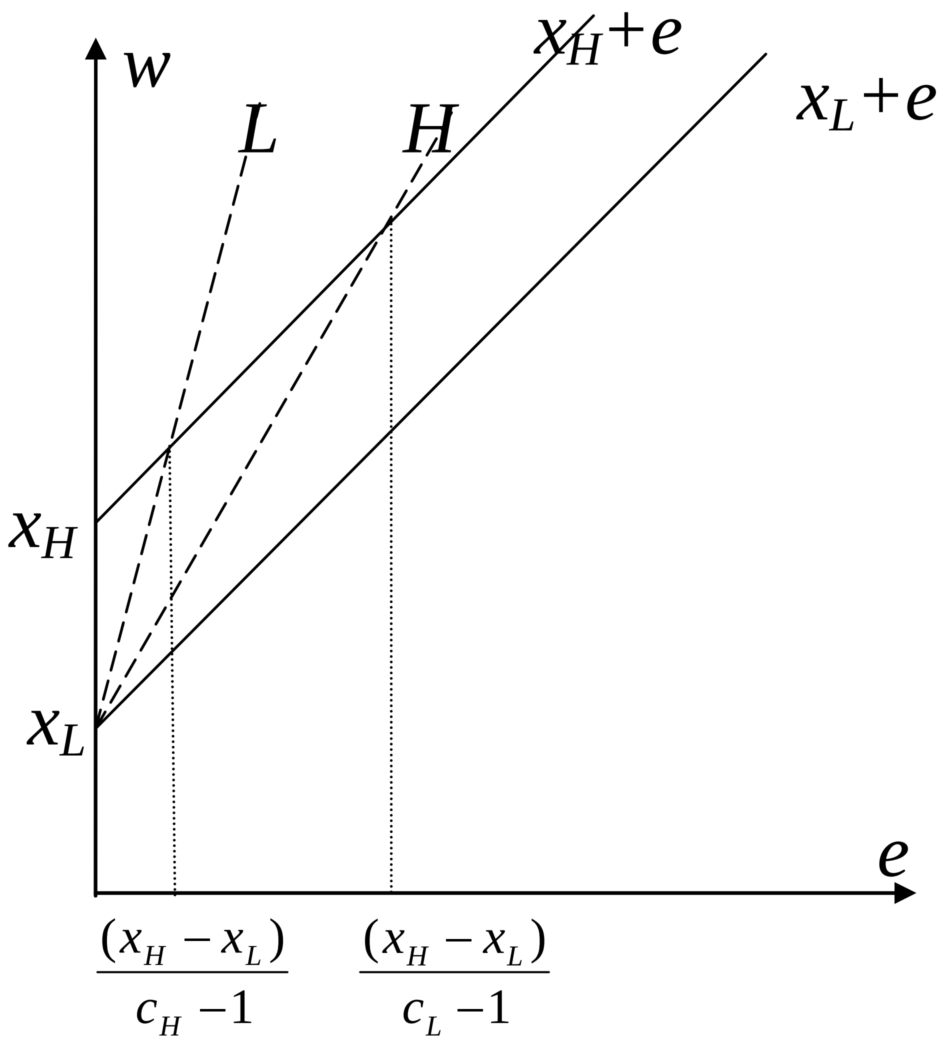
- he gets lower payoff in this information set than *in equilibrium*, ***no matter what are the beliefs in that set***

Then the belief in that information set must assign probability 0 to that type (unless the statement applies to all other types as well).

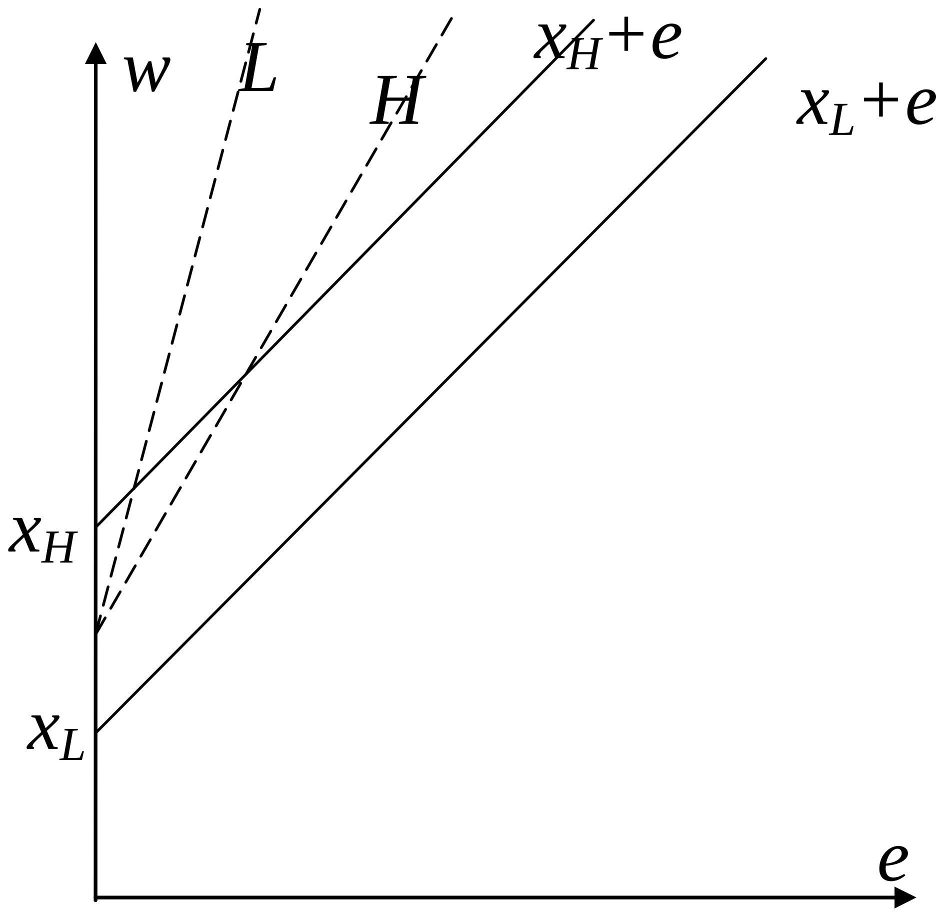
Intuitive criterion:

Informally speaking, if some type could never benefit from a given deviation (no matter what are the beliefs) but another type could, then one must assign probability 0 to the former type.





Intuitive Criterion in Education Signaling



Intuitive Criterion in Education Signaling (pooling)