Education Signaling Game (slightly altered)

Optional reading: MWG 13A, B, C; Kreps Ch. 17

- 2 types: L and H
- cost of education: $c_L(e)$ and $c_H(e)$ with

$c_L' > c_H' > 0$, $c_L'' > 0$ and $c_H'' > 0$

- productivities $x_L + e$ and $x_H + e$

Assume that $c_L'(0) < 1$ and $c_H'(0) < 1$, but $c_L'(e) \to \infty$ and $c_H'(e) \to \infty$ as $e \to \infty$.

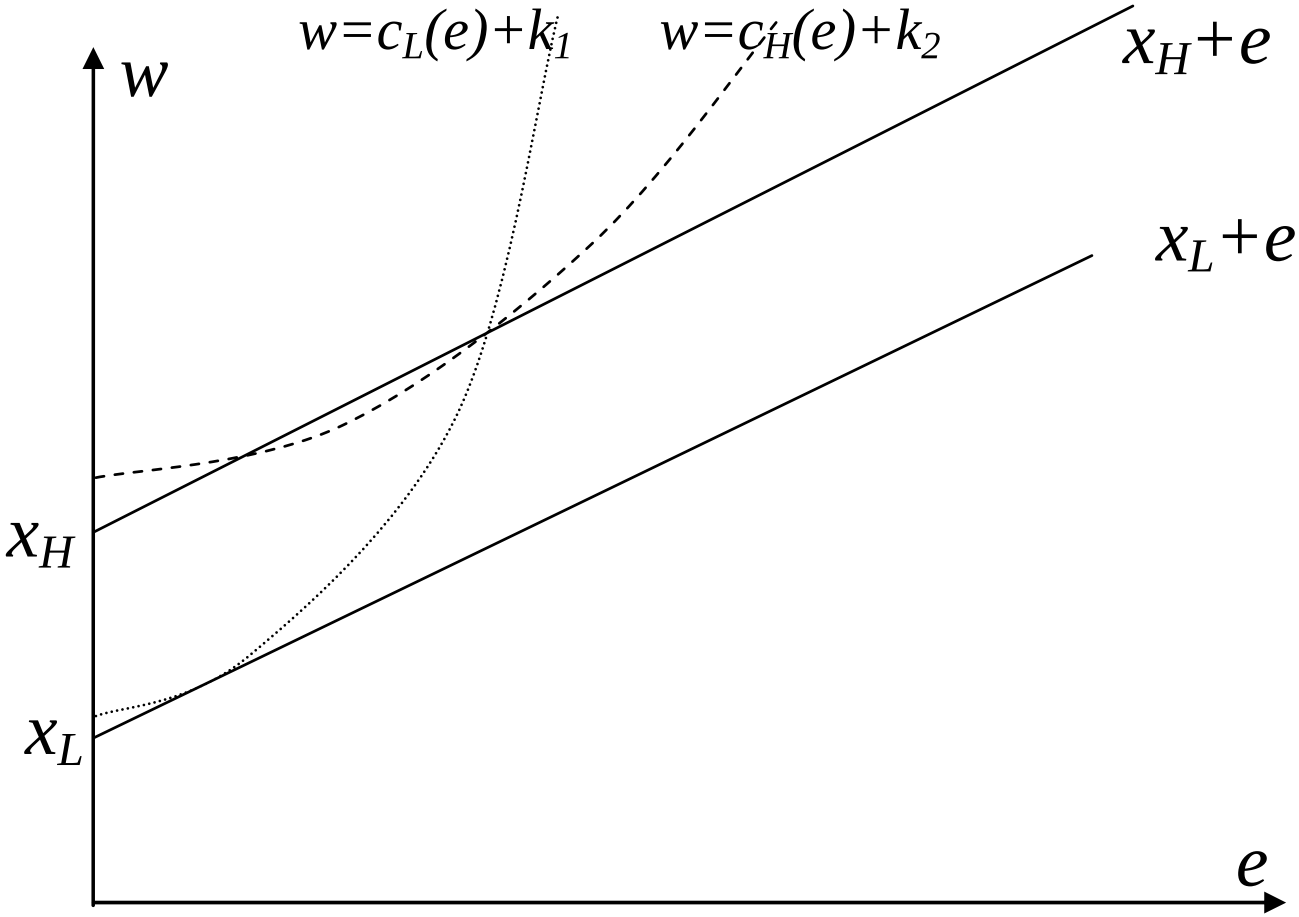
What condition defines the efficient education level?

Separating Equilibrium

- as before, type L must choose the efficient education level e_L, one that maximizes $x_L + e - c_L(e)$

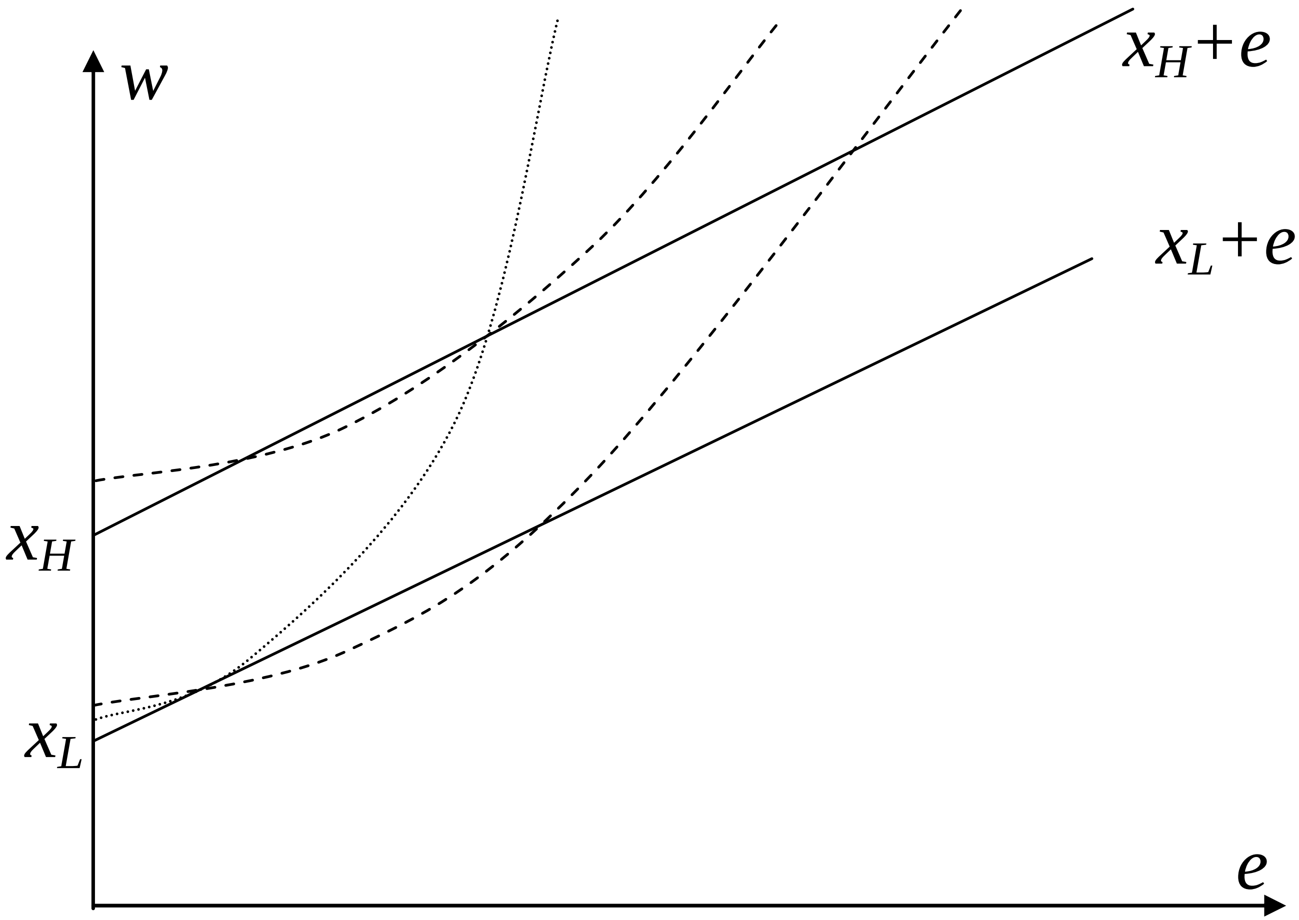
What is the argument behind this?

$$x_H + e_H - c_H(e_H) \geq x_L + e_L - c_H(e_L)$$
$$x_H + e_H - c_L(e_H) \leq x_L + e_L - c_L(e_L)$$



Because $c_L' > c_H' > 0$, type L's indifference curve is always steeper than type H's.

Separating Equilibria and Intuitive Criterion.



Adverse Selection:

- One seller w/ a good used car w/ prob. q or a “lemon” w/ prob. $1 - q$
- lemon and good cars have values $B < G$ to the seller
- many buyers with values $3/2\ B$ and $3/2\ G$ depending on the type of a car, who do not know the type of car
- buyers simultaneously bid a price
- the seller selects the highest bidder and sells at that price or refuses all bids and keeps the car

What happens in equilibrium?

Possibility 1:

- All buyers bid expected value 3/2 $B(1-q)$ + 3/2 Gq
- Both types of sellers accept

Under what condition is this an equilibrium?

Possibility 2 (Adverse Selection):

- All buyers bid 3/2 B
- The seller accepts only if the car is a lemon

Under what condition is this an equilibrium?

Are any other equilibria possible?

Application: Investment Game.

- Two types of firms with value L (prob. *p*) and H
- Profitable business opportunity:
  - o requires investment I
  - o generates return b + I
- Investment funds must be raised from equity

  if a firm sells equity fraction α, the fraction is worth

  $(L + b + I)\,\alpha$ or $(H + b + I)\,\alpha$

  depending on the firm type
- For all $\alpha > 0$, market forms belief $\mu(\alpha) = \mathrm{Prob}(H|\alpha)$ and pays $(\mu(\alpha)H + (1-\mu(\alpha))\,L + b + I)\,\alpha$

Investment Game.

<u>Summary</u>: L w/ prob. p and H w/ prob. 1-p
Business opportunity: investment I, return b + I
If equity share $\alpha > 0$ is offered, the market pays

$$(\mu(\alpha)H + (1-\mu(\alpha))\,L + b + I)\,\alpha \quad (*)$$

A firm may choose to raise I through equity (choose α s.t. (*) = I), or not invest

Payoffs: a firm cares about the value to *old* shareholders:

Type H: H vs. $(1-\alpha)(H + b + I)$
Type L: L vs. $(1-\alpha)(L + b + I)$

Denote by α^* the lowest value of α such that

$$(\mu(\alpha)H + (1-\mu(\alpha))\, L + b + I)\, \alpha = I$$

Under what condition is there a pooling equilibrium, i.e. both firms invest?

$\alpha = ?$

Recall: L w/ prob. *p* and H w/ prob. 1-*p*

Payoffs for Type H: H vs. $(1-\alpha^*)$ (H + b + I)

Type L: L vs. $(1-\alpha^*)$ (L + b + I)

Separating Equilibrium:

Only one type of a firm will invest. Which one?

Under what condition is there a separating equilibrium?

$\alpha = ?$

Recall: L w/ prob. *p* and H w/ prob. 1-*p*

Payoffs for Type H: H vs. $(1-\alpha^*)\ (H + b + I)$

Type L: L vs. $(1-\alpha^*)\ (L + b + I)$

$(\mu(\alpha^*)H + (1-\mu(\alpha^*))\ L + b + I)\ \alpha = I$