Ec 136, Financial Economics

Lecture 19

November 10

Outline for today

1. Mean-stdev frontier with many risky assets

2. Optimal portfolio with many risky assets

3. CAPM: introduction

www.econ.berkeley.edu/~szeidl/ec136/ec136index.htm

Readings: BKM Chapters 6, 7, 8.1-8.2 (6 and 7.1, 7.2 in editions 7&8)

Problem set 6: due November 12, Thurs, in class.

1. Frontier w/ many risky assets

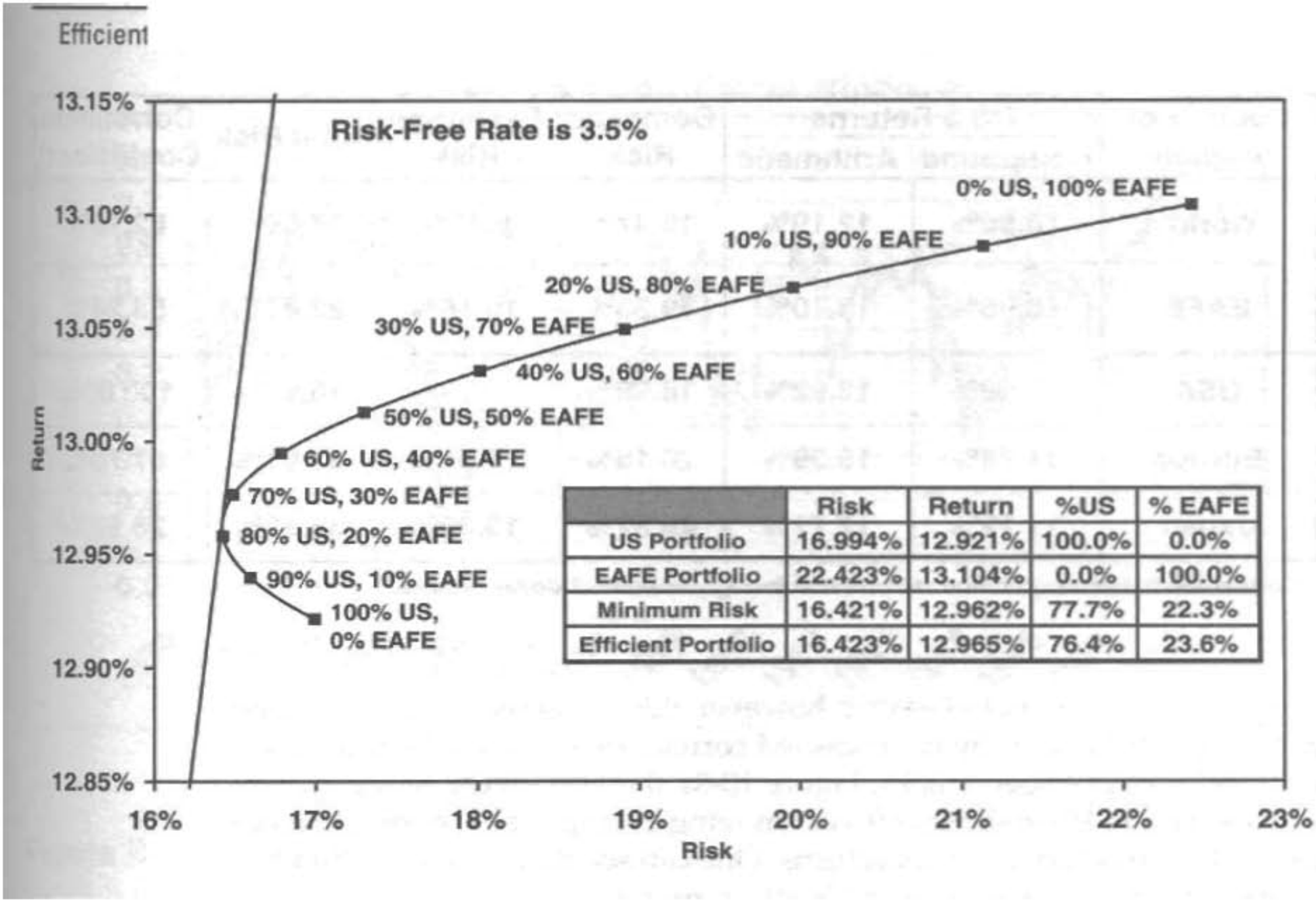
- Two risky assets $\Longrightarrow$ cup-shaped region.
  - Similar shape with may risky assets.

- Now add safe asset:
  - Combining R_f with risky portfolio R_p, can travel along straight line connecting them
  - Slope is Sharpe ratio:

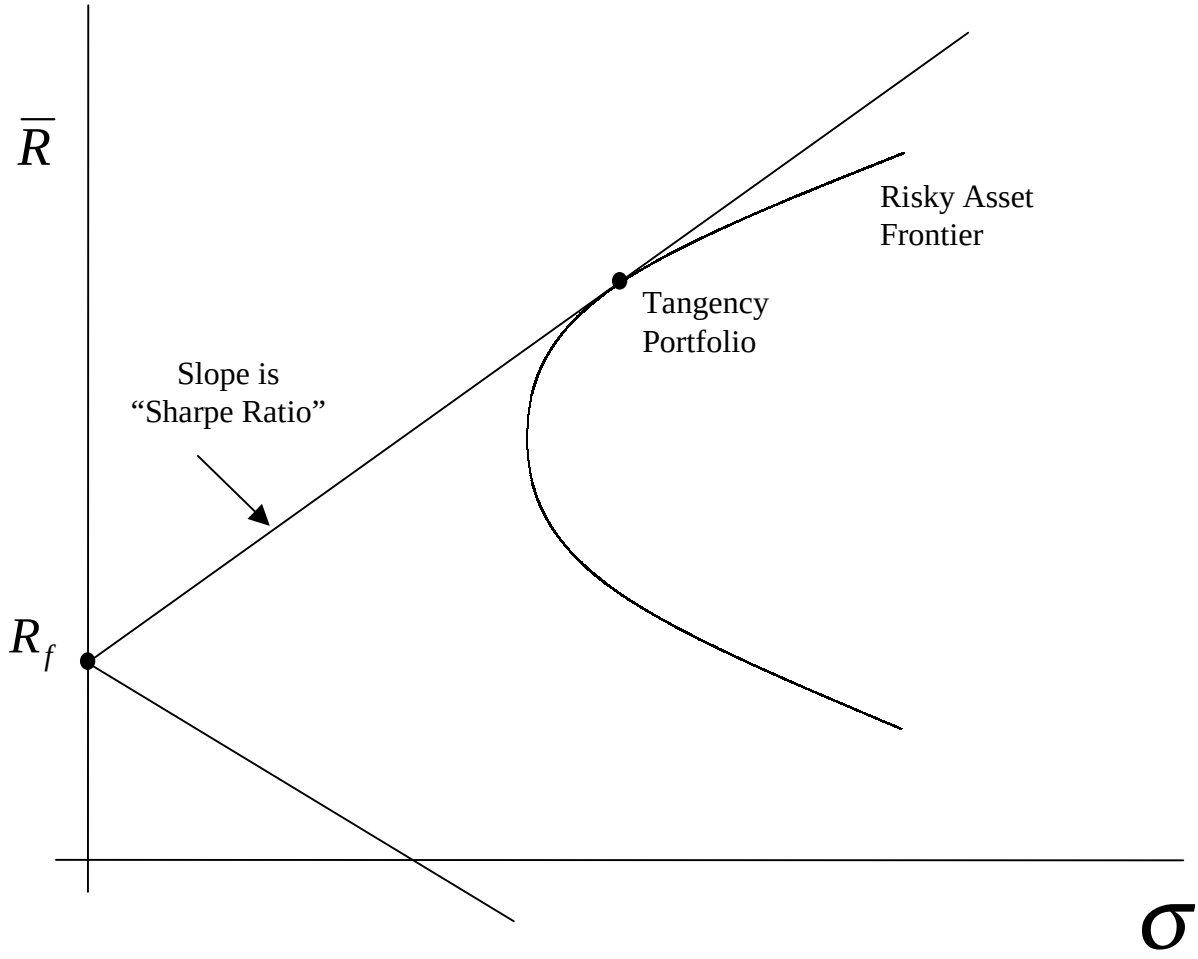
$$S_p = \frac{\mathsf{E}R_p - R_f}{\sigma_p}$$

- **Tangency portfolio:** where best straight line meets mean-stdev frontier of risky assets.
  - Has highest Sharpe-ratio

- *Global* efficient frontier: line connecting R_f and tangency portfolio.



	Risk	Return	%US	% EAFE
US Portfolio	16.994%	12.921%	100.0%	0.0%
EAFE Portfolio	22.423%	13.104%	0.0%	100.0%
Minimum Risk	16.421%	12.962%	77.7%	22.3%
Efficient Portfolio	16.423%	12.965%	76.4%	23.6%


$\overline{R}$
Risky Asset
Frontier
Tangency
Portfolio
Slope is
“Sharpe Ratio”
R_f
σ

2. Optimal portfolio

- Optimal portfolio: find best combination of R_f and T just like with only two assets

$$w_1 = \frac{\mathsf{E}R_T - R_f}{A\sigma_T^2}.$$

- Principle of participation: if $\mathsf{E}R_T - R_f > 0$, should hold some stocks.

- Implementing mean-variance analysis:

  1. Use securities data to construct risky mean-variance frontier.

  2. Find tangency portfolio.

  3. Find optimal mix of tangency portfolio and riskfree asset.

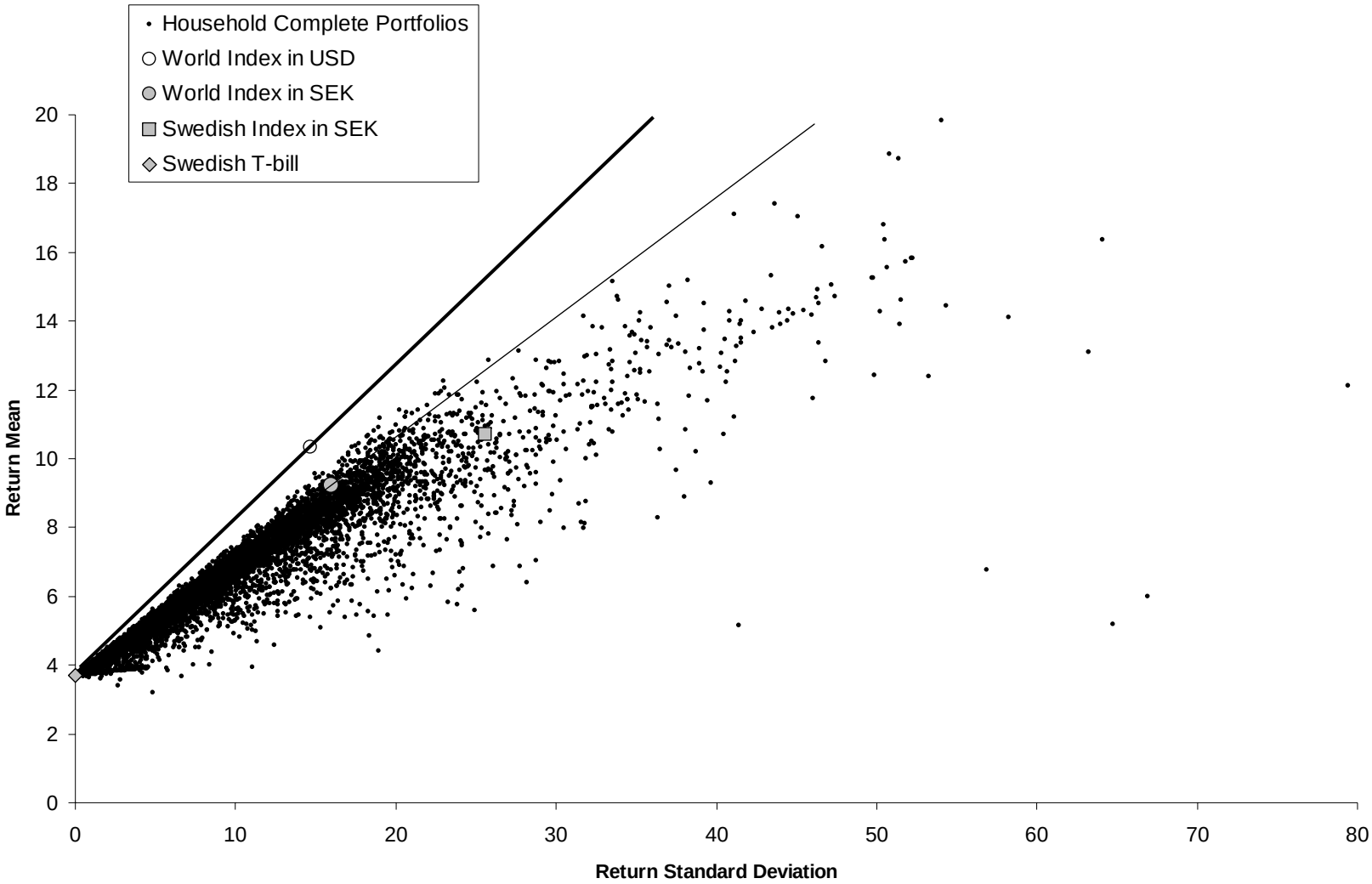
Mutual fund theorem

- **Mutual fund thm:** All efficient portfolios are combinations of tangency portfolio and riskfree asset.

- All investors should hold risky assets in the same proportions.
  - Conservative investors should dilute risky asset holdings with cash, but should not change composition of those holdings

- Consistent with evidence from Sweden?

b. Complete Portfolios



Mean-var analysis in practice

- Commonly used (software packages)
  - Build on matrix formulas generalizing our math
- Difficulties:
  1. Need to estimate average returns
     - Imprecise, may move over time
  2. Must estimate N^2 covariances
  3. With many assets, historical data often suggests some portfolio is close to riskless
     - Optimizer will propose large leveraged bet
- Difficulties motivated search for shortcut methods to find optimal portfolios $\Longrightarrow$ CAPM

3. CAPM

- CAPM states that market portfolio is efficient.
  - Market portfolio = portfolio of all risky assets.

- CAPM assumptions:
  - Investors care about returns measured over one period.
  - There are no nontraded assets or transactions costs.
  - All investors are mean-variance optimizers.
  - All investors perceive the same means, variances and covariances for returns.

Derivation of CAPM

- These assumptions imply that

  1. All investors work with the same mean-stdev diagram.

  2. All investors hold mean-var efficient portfolio.

  3. Mutual fund thm $\Longrightarrow$ all investors hold risky assets in the same proportions to one another.

  4. Demand $=$ supply, so all investors must hold market portfolio, i.e., value-weighted index that contains all risky assets in proportion to their market value.

  5. Market portfolio is mean-var efficient.

- **CAPM**: market portfolio is mean-variance efficient.

Implications of CAPM

- If CAPM holds, no need to perform mean-var analysis.
  - An investor can free-ride on analyses of other sophisticated investors
  - Use market portfolio as optimal mutual fund of risky assets.
- In practice, funds indexed to S&P 500 or Wilshire 5000 can be used as proxies for market portfolio.
- Do households hold efficient portfolios?
  - Some make mistakes, most invest efficiently.