

**ECONOMICS 136  
SAMPLE FINAL EXAM A  
SUGGESTED SOLUTIONS**

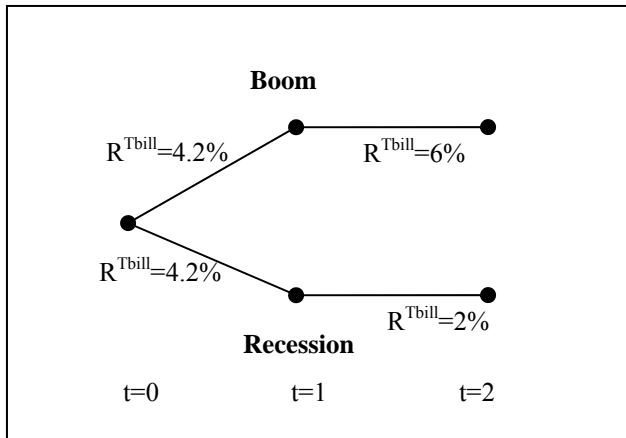
**PART I**

1. True. If CAPM holds then the market portfolio is the same as the tangency portfolio, and no portfolio has a higher Sharpe-ratio than the tangency portfolio.
2. True. As time passes, the time to maturity for any given bond decreases (since the date of maturity is unchanged); with an upward sloping yield curve, this means that the yield of the bond is falling, and hence its price is rising. This is called riding the yield curve.
3. True. Intuitively, the second option “does more” than the first option: it has a lower exercise price and expires later, and hence should have a higher price than the first option. Formally, you can “pretend” that the second option has a strike price of \$12 and expires in June, and then you have two identical options with different prices – selling the first one and buying the second one is an arbitrage opportunity.
4. False. LSV argue that investors have rushed to buy growth stocks, and as a result they are currently overvalued. Eventually, this mispricing will be corrected by low subsequent returns, which is confirmed in the data where growth stocks earn low returns.
5. True. One implication of the mutual fund theorem is that the composition of risky assets should be equal in all investors' portfolios. Here, the ratio of growth to value stocks is 1/3 for Congyan and 1/4 for Dan, which are not equal.
6. False. The value effect says that stocks with high book-to-market ratios earn higher returns. But the weak form of the EMH only requires that one cannot find portfolios based on price data that earn positive abnormal returns; to form a portfolio of value stocks, you also need accounting data on the book value of firms.

**PART II**

**1. Portfolio choice** (24 points, 4 each)

- (a) The total rate of return between  $t=0$  and  $t=2$  if there was a boom is  $1.042 \cdot 1.06 - 1 = 10.5\%$ . If there was a recession, then the total rate of return is  $1.042 \cdot 1.02 - 1 = 6.3\%$ .



(b) The price of the Treasury bond is the discounted value of its future payment. Since both the T-bond and the T-bill are riskfree between  $t=1$  and  $t=2$ , the appropriate discount rate is the T-bill rate, which is 6% if the economy is in a boom, yielding  $P_1=100/1.06=\$94.34$ . Similarly, if the economy is in a recession, we obtain  $P_1=100/1.02=\$98.04$ .

(c) The return on the Treasury bond between  $t=0$  and  $t=1$  is  $94.34/92-1=2.5\%$  if the economy enters a boom, and  $98.04/92-1=6.6\%$  if the economy enters a recession.

(d) Mean return for Treasury bond:  $0.5 \cdot 2.5\% + 0.5 \cdot 6.6\% = 0.0455 = 4.55\%$

Variance:  $0.5 \cdot (0.025 - 0.0455)^2 + 0.5 \cdot (0.066 - 0.0455)^2 = 0.00042$

Standard deviation: **0.0205**.

The Treasury bill is riskfree between  $t=0$  and  $t=1$ , with a mean return of **4.2%**.

The optimal portfolio share of the Treasury bond is

$$W_{\text{bond}} = (ER_{\text{bond}} - R_{\text{bill}}) / A\sigma_{\text{bond}}^2 = (0.0455 - 0.042) / (10 \cdot 0.00042) = 83.33\%.$$

Thus the optimal portfolio consists of **83.33%** in T-bonds and **16.67%** in T-bills for the short-term investor.

(e)

Mean and variance of the money market fund over 2 periods:

Mean return:  $0.5 \cdot 10.5\% + 0.5 \cdot 6.3\% = 0.084 = 8.4\%$

Variance:  $0.5 \cdot (0.105 - 0.084)^2 + 0.5 \cdot (0.063 - 0.084)^2 = 0.000441$ .

Standard deviation: **0.021**.

The T-bond is riskfree over the time period between  $t=0$  and  $t=2$ , and has a mean return of  $100/92-1=8.7\%$ . Thus the Treasury bond strategy is better: it is riskfree and has a higher expected return than the money market fund. Any mean-variance optimizer with positive  $A$  would choose strategy 1 over strategy 2.

(f) The advice is sound. T-bills are safe in the short term, but as the problem demonstrates, can be risky in the long term. On the other hand, long term bonds are risky in the short term but safe in the long term. For a long term investor, long term bonds are likely to be safe; to avoid inflation risk, one might choose to invest in inflation-protected Treasury bonds. Long-term investors should hold long-term bonds.

## 2. Capital budgeting (18 points, 3 each)

(a) This is inappropriate because the manager discounts the future dividends using the riskfree return of 4%. But ABC stock is risky (has a nonzero covariance with the market return), and hence requires a risk-adjusted discount rate.

(b) The beta of ABC is  $\beta = 0.06 / 0.2^2 = 1.5$ . By the CAPM equation the expected return on ABC is  $ER = 4\% + 1.5 * (12\% - 4\%) = 16\%$ .

(c) By the Gordon growth model,  $P = D / (ER - G) = 10 / (0.16 - 0.02) = \$71.43$ .

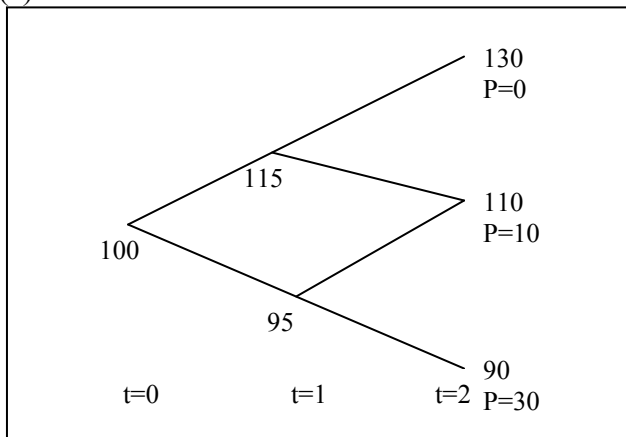
(d) Yes. It seems that ABC is a value stock. We know that value stocks on average earn higher returns than what is predicted by CAPM. Given that value stocks have higher expected returns, we should use a higher number for ER in the Gordon growth formula, which can explain why the true price of \$50 is lower than what we found in part (c).

(e) The Fama-French model implies that the expected return of ABC must satisfy  $ER - R_f = (1.5) * (12\% - 4\%) + 1.5 * (4\%) + 0 = 18\%$ , so that  $ER = 22\%$ . This is higher than the 16% obtained in (b), because the Fama-French model accounts for the value effect.

(f) The Gordon model implies that  $P = D / (ER - G) = 10 / (0.22 - 0.02) = \$50$ . Thus the Fama-French model does justify the market price in this example.

## 3. Derivatives

(a)



(b) To make the portfolio a replicating portfolio for the put option, we need its payoffs to match the put's payoffs when the stock goes up and when it goes down:  $130x + y = 0$  and  $110x + y = 10$ . Thus, this replicating portfolio consists of  $x = -0.5$  and  $y = 65$ , which has price  $115 * (-0.5) + 65 = 7.5$ . Since this portfolio has the same payoffs as the put option in all future states, by no arbitrage it must have the same price as the put option at this event.

(c) Similarly,  $110x+y=10$  and  $90x+y=30$ , thus  $x=-1$  and  $y=120$ . The price of the option must be  $P_1 = -95+120=25$ .

(d) Similarly,  $115x+y=7.5$  and  $95x+y=25$ , thus  $x=-0.875$  and  $y=108.125$ . The price of this portfolio = price of the put option  $P_0 = 100*(-0.875)+108.125=20.625$ .

(e) The expected net return is  $(7.5/2+25/2)/20.625-1=-0.21$  or  $-21\%$ . The put has a negative expected return, which is lower than the riskfree rate of zero. This is because the put provides insurance. If CAPM holds, the put must have a negative beta.

(f) The expected market return is  $(115/2+95/2)-1=0.05$  or  $5\%$ . The variance of the return is  $((0.15-0.05)^2+(-0.05-0.05)^2)/2=0.01$ . The covariance of the market with the put is  $(0.1*7.5/20.625-0.1*25/20.625)/2=-0.042$ .

(g) The beta of the put is  $-0.042/0.01=-4.2$ , which is indeed negative. The predicted expected return is  $-4.2*0.05=-0.21$  or  $-21\%$ . This is the same number we got in (e). Thus CAPM holds in this example.

### **PART III - ESSAY**

#### **Mutual funds**

a) Passively managed funds on average earn higher returns. This is in part due to fee differences: passive funds have lower fees than active funds. Active funds charge higher fees partly because active management and stock picking requires more resources. The lower average returns of active funds are also due to transactions costs: active funds tend to trade substantially more than passive funds. Based on these facts, it seems that the investment strategy recommended by your friend is not as good as simply investing in a passive index fund.

b) In a population of many fund managers, just by luck there will be some who perform better than the market. Thus, when you see good fund performance, the question is always whether it is due to luck, or due to a smart manager. If it is due to a smart manager, you expect the high positive returns to persist over time. There is some evidence that fund returns are persistent, but most it is coming from persistently poor returns. Overall, the evidence suggests that much of the good performance is due to luck, but that does not exclude the possibility that investors like Buffett and Soros earned high returns because of their ability of choosing the right investments.

c) Survivorship bias. The problem is that some of the funds that didn't do so well between 1995 and 2004 were closed down. The chart, which shows only funds that existed in 2005 is thus showing funds that generally performed better than average during 1995-2005; but in 1995, you cannot predict who are the funds that will not be closed down before 2005. The chart thus inflates the performance of funds. The correct chart to look at is one that shows the performance of all funds that existed in any given year between 1995 and 2005.