Department of Economics **Fall 2009**
University of California, Berkeley

ECONOMICS 136
SAMPLE FINAL EXAM B
SUGGESTED SOLUTIONS

PART I

1. True. You can buy the put option at a cost of $1.60 and exercise immediately to earn $3. This is a riskfree investment opportunity that has zero cost and generates an immediate profit of $1.40.

2. False. For example, in a CAPM equilibrium, two securities can have the same standard deviation and different betas. These two securities will have different expected returns.

3. False. While the value effect is indeed not explained by CAPM, the empirical fact is that stocks with high book-to-market ratios, i.e., value stocks, have low betas and high average returns.

4. True. If CAPM holds than the market portfolio is the same as the tangency portfolio, and no portfolio has a higher Sharpe-ratio than the tangency portfolio.

5. False. The optimal share of the tangency portfolio in the portfolio of a mean-variance optimizer is $w_T=(ER_T-R_f)/ A\sigma^2_T$ which is positive for any A>0. Hence even a highly risk-averse mean-variance optimizer will choose to hold some stocks in her portfolio.

6. False. In a CAPM economy, all investors hold the market portfolio and the riskfree asset; and the market portfolio is a portfolio of all risky assets, including those with an expected return below the riskfree rate. Such assets are useful because they have a negative beta, which means that they provide insurance against fluctuations in the rest of the portfolio.

7. True. The payoff function of this strategy is an “inverted V,” which has low payoffs if the stock price moves away significantly in either direction from the current price. Note, this strategy always has a non-positive payoff in the future, but it provides revenue to the investor on the day of the sale.

PART II

1. Capital budgeting (15 points, 3 each)

a) The beta of ABC is β =0.18/0.3^2=2. By the CAPM equation the expected return on ABC is ER=2%+2*(12%-2%)=22%.

b) By the Gordon growth model, P=D/(ER-G)=10/(0.22-0.12)=$100.

c) If G=7% then P=D/(ER-G)=10/(0.22-0.7)=10/0.15=$66.6.

If G=17% then P=D/(ER-G)=10/(0.22-0.17)=10/0.05=$200.

Given that these two possibilities have equal probability, and that the realization of G is uncorrelated with the return on the market portfolio, the price per share should be simply P=0.5*$66.6+0.5*$200=$133.3.

d) The answer to c) is higher. Thus when there is uncertainty about dividend growth, the value of the company is higher than when dividend growth is known with certainty. The intuition is that uncertainty allows for the possibility of very high dividend growth, which leads to high future dividends and a correspondingly high stock price with positive probability. More formally, the Gordon formula is a convex function in G, thus uncertainty about G increases it's expected value.

e) The example suggests that this argument is plausible. Perhaps investors thought that technology companies have a small but positive chance of becoming extremely productive. Then, by the logic of the example, technology companies should have a high price.

2. Portfolio choice (16 points, 4 each)

(a) Mean and variance over 1 period:

Mean return: $0.5.15+0.5\cdot(-0.05)=0.05=5\%$

Variance: $0.5\cdot(0.15-0.05)^2+0.5\cdot(-0.05-0.05)^2=0.01$.

Mean and variance over 2 periods:

Mean return: $0.25.30+0.5\cdot(0.1)+0.25\cdot(-0.10)=0.1=10\%$

Variance: $0.25\cdot(0.30-0.10)^2+0.5\cdot(0.10-0.10)^2+0.25\cdot(-0.1-0.1)^2=0.02$.

(b) For a one-period investment, the optimal portfolio share of the index is

$w_I=(ER_{01}-R_{f,01})/A\sigma_{01}^2=(0.05-0)/0.1=50\%$.

So the optimal portfolio consists of 50% riskfree and 50% stocks.

For a two-period investment, the optimal portfolio share of the index is

$w_I=(ER_{02}-R_{f,02})/A\sigma_{02}^2=(0.1-0)/0.2=50\%$.

So the optimal portfolio again consists of 50% riskfree and 50% stocks.

c) The probabilities of the various index values at t=2 are: 130 with probability 1/8, 110 with probability ¾, 90 with probability 1/8. The mean and variance of the index return over 2 periods are:

Mean return: $1/8\cdot0.30+3/4\cdot(0.1)+1/8\cdot(-0.10)=0.1=10\%$

Variance: $1/8\cdot(0.30-0.10)^2+3/4\cdot(0.10-0.10)^2+1/8\cdot(-0.1-0.1)^2=0.01$.

Thus the mean return is the same as earlier, but the variance is smaller. The intuition is that following a price increase the price is more likely to fall, and following a price decrease the price is more likely to rise. Because of this mean-reversion in returns, the two period return is less risky here than it was in part (b).

The optimal portfolio share of the index is

$w_I=(ER_{02}-R_{f,02})/A\sigma_{02}^2=(0.1-0)/0.1=100\%$

which is higher than the corresponding answer in part (b).

(d) If stock returns exhibit mean-reversion, as in part (c), then the advice of financial analysts is sound. This is because mean reversion reduces the risk of stocks over long horizons, making them more attractive for long-term investors. If stock returns do not exhibit mean-reversion, as in parts (a) and (b), then the optimal portfolio is the same for a short-term investor as it is for a long-term investor.

3. Options (24 points, 4 each)

a)

b) Replication requires

130x+y=30

110x+y=10.

Solving, x=1 and y=-100. The price of this portfolio at t=1 is 115x+y=$15.

c) Replication requires

110x+y=10

90x+y=0.

Solving, x=0.5 and y=-45. The price of this portfolio at t=1 is 95x+y=$2.5.

d) Replication requires

115x+y=15

95x+y=2.5.

Solving, x=0.625 and y=-56.875. The price of this portfolio, which is also the call price, equals 100x+y=$5.625.

e) By put-call parity, the put price plus the stock price ($100) must equal the call price ($5.625) plus the discounted exercise price ($100). Thus the put price must be $5.625. But the actual put price, $5.425, is lower. Hence there is an arbitrage opportunity. To exploit it, buy a put and a share of the stock for $5.425+$100, and sell the call and the riskfree asset (i.e., borrow $100) for a total of $5.625+$100. Then today you pocket $0.2, and at T=2, your two positions exactly cancel, thus you neither gain nor lose money. This is an arbitrage strategy.

f) We didn't use the probabilities in deriving the results in parts a)-e). Hence changing the probabilities will not affect the call and the put price. The intuition is that in an efficient market, any changes in the probabilities of different outcomes should be incorporated in the stock price at date zero. Thus, once we know the stock price (or more generally, the binomial tree), we don't need to know the probabilities associated with the outcomes to compute the option price.

4. Derivatives (9 points, 3 each)

a) The portfolio of buying a call and selling a put has a payoff function which is an upward-sloping 45-degree line, crossing the horizontal axis at the point X. The price of this portfolio is the call price minus the put price. By assumption, these two prices are equal, hence the portfolio price is zero.

b) The payoff function of a long forward position is also an upward sloping 45-degree line. This line crosses the horizontal axis at the point F^T.

c) Both portfolios have a zero price today. Both portfolios have payoff functions that are upward sloping 45-degree lines. Unless these two lines are exactly on top of each other, there exists an arbitrage opportunity: you long the portfolio of the line above, and short the portfolio of the line below. Hence the two lines must be identical. This means that $X = F^T$. We know from the spot-futures parity that F^T=\$40*(1.05)=42. Hence X=\$42.

PART III – ESSAY (15 points)

The value effect

a) Value stocks are stocks that have high book-to-market ratios, while growth stocks have low book-to-market ratios. The value effect is that historically, value stocks have outperformed growth stocks as well as the market by a considerable margin.

b) No. The value effect would be consistent with CAPM if value stocks had high betas. But in the data, value stocks have low betas and high expected returns, which is inconsistent with the expected return-beta representation.

c) The Fama-French three factor model explains the value effect using the risk-return trade-off. In that model, value stocks appear risky because they have a high beta on the HML factor. In contrast, growth stocks appear less risky because they have a low (negative) beta on the HML factor. This is partly by construction: HML is the excess return of a portfolio of value stocks over a portfolio of growth stocks. Hence, by design, each value stock will have a high, positive beta with respect to HML.

d) The Fama-French model also explains the size effect, namely that small stocks have done better than large stocks, and reversal, namely that stocks that were winners in the past 3-5 years tend to have poor subsequent performance, and stocks that were losers in the past 3-5 years tend to perform well subsequently.

e) Investor psychology. According to this view, investors get excited about growth (glamour) stocks and bid up their prices. These companies become overvalued in the process of becoming growth stocks. Since they are overvalued, they have low subsequent returns. Meanwhile, investors neglect value stocks, which become temporarily underpriced, explaining their high subsequent returns.