

**ECONOMICS 136  
SAMPLE FINAL EXAM C  
SUGGESTED SOLUTIONS**

**PART I**

1. True. One arbitrage strategy is to buy the stock and sell the call. This earns \$0.30 today, and you have your option covered: if it is exercised at a future date, you can just sell the other party the ABC stock that you had purchased.
2. False. If the two assets are negatively correlated, the portfolio variance can be smaller than the variance of either, or even zero. For example, the head and tail assets discussed in class, if combined in a portfolio, would have zero variance.
3. False. The CAPM equation says that XYZ will have expected return below the riskfree rate; but the expected return might still be positive, if the beta of XYZ is not too negative.
4. True. Prepayment typically occurs when interest rates are low. This means that the mortgage investor receives cash exactly when his/her reinvestment opportunities are poor.
5. False. It would be optimal to exercise immediately a put option of a company that is bankrupt and has a share price of zero. This is because the put payoff cannot increase any further, and a dollar now is better than a dollar later.
6. True. We know that a no-arbitrage argument implies that the call price should always exceed the payoff from immediate exercise for a non-dividend-paying stock. For completeness, the arbitrage strategy would be to buy the call, short the stock and invest  $X/(1+R_f)$  in a riskfree asset: this has a positive payoff today, and on the date of expiration the option is in the money exactly if the short is losing more than the value of the safe investment.

**PART II**

**1. Market timing**

(a) See the figure. The net return between  $t=0$  and  $t=1$  is  $130/100-1=0.3$  or 30% if the index goes up and  $90/100-1=-0.1$  or -10% if the index goes down. The expected return is  $(0.3-0.1)/2=0.1=10\%$ .

(b) The expected return between  $t=1$  and  $t=2$  if the index is up at  $t=1$  is  $(160*(1/4)+120*3/4)/130-1=0\%$ . This is lower than the expected return of 10% between  $t=0$  and  $t=1$ .

(c) The probability of reaching each node is  $1/8, 3/8, 3/8$  and  $1/8$ . The realized return for these nodes is  $160/100-1=0.6=60\%$ ,  $120/100-1=0.2=20\%$ , and  $80/100-1=-0.2=-20\%$ .

Expected return:  $(1/8)*0.6+(6/8)*0.2+1/8*(-0.2)=0.2=20\%$ .

Variance of return:  $(1/8)*(0.6-0.2)^2+(6/8)*(0.2-0.2)^2+(1/8)*(-0.2-0.2)^2=2/8*(0.4^2)=0.04$ .

(d) The optimal share of stocks is  $(0.2-0)/(5*0.04)=100\%$ . The portfolio return in the four final states is therefore  $0.6=60\%$ ,  $0.2=20\%$ ,  $0.2=20\%$ , and  $-0.2=-20\%$ . The mean portfolio return is the same as the mean index return of  $0.2=20\%$  and the variance is  $0.04$ .

(e) \$100 becomes \$130 if the market goes up at  $t=1$ , and remains \$130 at  $t=2$  since it is reinvested in the risk-free asset. In the other two states, the portfolio value will continue to be \$120 and \$80. The net returns in the four nodes are thus  $0.3=30\%$ ,  $0.3=30\%$ ,  $0.2=20\%$  and finally  $-0.2=-20\%$ .

(e) The expected return is  $(1/2)*0.3+(3/8)*0.2+(1/8)*(-0.2)=0.2$ , which is the same as in (d). The variance of the portfolio return is  $(1/2)*0.1^2+3/8*0+1/8*(-0.4)^2=0.025$ .

We find the same expected return but lower variance: a risk-averse investor would thus prefer strategy (e). Market timing can benefit investors.

(f) Valuation ratios like the price/earnings ratio predict subsequent returns. This exercise suggests that investors should reduce their stockholdings when expected returns are predicted to be low, and increase them when expected returns are predicted to be high.

## 2. Out-of-the-money puts

(a) The expected gross market return is  $0.75*1.2+0.25*0.8=1.1$ , so the net expected return is  $10\%$ . The variance of the market return is  $0.75*(0.1)^2+0.25*0.3^2=0.03$ .

(b) The put pays \$0 in the boom state and \$1 in the recession state. A replicating portfolio of  $x$  shares in the stock and  $y$  shares in the bond must satisfy

$$120x+y=0 \text{ and } 80x+y=1.$$

Solving this system of equations gives  $x=-1/40$  and  $y=3$ . The cost of this portfolio is just  $(-100/40)+3=0.5$ . This is the price of the put.

(c) The realized return of the put is  $-100\%$  in a boom, as all money is lost. In a recession the gross return is  $1/0.5=2$ , so the net return is  $100\%$ . The expected gross return is  $0.75*0+0.25*2=0.5$ . Thus the expected net return is  $-50\%$ , which is lower than the riskfree rate of zero.

(d) The covariance of the market and the put return is  $0.75*0.1*0+0.25*(-0.3)*2=-0.15$ . The beta of the put is therefore  $-0.15/0.03=-5$ . Beta has a negative sign because puts are insurance: they pay off exactly when the market is doing poorly.

(e) Given that the riskfree rate is zero, the expected net return of the put according to CAPM should be  $-5*(0.1-0)=-0.5$  or  $-50\%$ . This is the same as the answer we got in (c): CAPM does appear to hold in this example.

(f) Selling 200 puts generates \$100. In a boom the payoff is thus \$200; in a recession, the payoff is  $\$200 - \$200 = 0$ , because all the puts are exercised. The probability of losing money is only 25%, but if you do lose money, you lose all of it.

(g) The reasoning is not credible. The hedge fund may be selling out-of-the-money puts. For example, the strategy of part (f) earns high returns with 75% probability, but loses everything with the remaining 25% probability.

### 3. Shorts

(a) The forward price is  $F_0^T = (1 + R_f)S_0 = 1.1 * 100 = \$110$ .

(b) See the figure. The forward payoff is a downward sloping 45 degree line that crosses the horizontal axis at \$110.

(c) This portfolio has the same payoff line in the diagram. To see why, note that this portfolio loses a dollar for each dollar increase of the underlying because of the short; and it earns a payoff of zero when the price of the underlying is \$110, because then the riskfree investment is just sufficient to buy back the stock (i.e., to close the short). Thus the two portfolios never differ in payoffs, and also have the same price of zero at  $t=0$ .

(d) Since futures are marked to market, if the price of the underlying moves against you, you may get a margin call, asking you to put up more capital; otherwise your broker may close your position.

## PART III - ESSAY

a) Valuation ratios normalize the stock price by some accounting measure of size or performance. Commonly used ratios include the price-earnings ratio, the dividend-price ratio and the book-to-market ratio.

b) The P/E of the US market has moved around over time, but eventually tended to return to its historic mean. If P/E is expected to return to its mean, then high values of P/E must predict either a subsequent fall in P or a subsequent rise in E. Conversely, low values of P/E must predict either a subsequent rise in P or a subsequent fall in E. Thus, P/E must either be negatively related to subsequent price growth or positively related to subsequent earnings growth.

c) In the data, P/E predicts subsequent price growth, and is unrelated to subsequent earnings. The events of this fall were consistent with this prediction: high P/E was followed by a fall in prices. The current P/E of the market is around, or slightly below, historic average. Thus P/E does not have a sharp prediction.

d) Across stocks, companies with high P/E (or low book-to-market), also known as growth stocks, earn low returns; while stocks with low P/E (or high book-to-market), also known as value stocks, earn high returns. This is the value effect.