Economics 136. Financial Economics

Sample midterm 1A, Fall 2009, Suggested solutions

1. True or false. (18 points, 6 each)

(i) False. The value of Vitamin stocks next year is $\$800 \cdot 0.92 = \$736$, but you owe \$315 to the bank, so your wealth is \$736-\$315=\$421. Your gross return is then 421/500=0.84 and hence your net return is -0.16 or -16%.

(ii) True. By put call parity, we should have $C_0 + X/(1 + R_f) = P_0 + S_0$, but $C_0 + X/(1 + R_f) = 3 + 35/1.05 = 36.33$, while $P_0 + S_0 = 2.5 + 33 = 35.5$. Since there are no transaction costs, we can make arbitrage profits by selling the first portfolio and buying the second.

(iii) False. Consider a one-year coupon bond with face value $F = 10,000$ and a single coupon payment of $C = \$1000$ on maturity. If the yield of this bond is $R = 5\%$ then it's price today equals $(F + C)/(1 + R) = 11000/1.05 = 10,476.19 > 10,000$. [More generally, the price is higher than the face value if the yield is lower than the coupon rate.]

2. Portfolios. (18 points, 6 each)

a) An *equal-weighted portfolio* has 1/2 weight in company 1 and 1/2 weight in company 2. Since you have \$10,000, you would invest \$5,000 in company 1 and \$5,000 in company 2. You buy $5,000/100 = 50$ shares of company 1. Similarly, buy $5,000/50 = 100$ shares of company 2.

Value-weighted portfolio. The value of company 1 is $10,000 \cdot 100 = \$1M$. The value of company 2 is $60,000 \cdot 50 = \$3M$. So the weight of company 1 in the portfolio is $1M/(1M + 3M) = 1/4$ and the weight of company 2 is 3/4. You would invest \$2,500 in company 1 and \$7,500 in company 2, which means buying $2,500/100 = 25$ shares of company 1 and $7,500/50 = 150$ shares of company 2.

b) The simple net return on company 1 is $R_1 = (150 - 100)/100 = 0.5$ or 50%. The simple net return on company 2 is $R_2 = (50-50)/50 = 0$. The portfolio return is computed as $R_p = w_1 R_1 + w_2 R_2$. For the equal-weighted portfolio, the return is $1/2 \cdot 0.5 + 1/2 \cdot 0 = 0.25$ or 25%. For the value-weighted portfolio, the return is $1/4 \cdot 0.5 + 3/4 \cdot 0 = 0.125$ or 12.5%. The portfolio return is higher for the equal-weighted portfolio because we invested a larger proportion of our budget in the asset with the higher return.

c) To keep the equal-weighted portfolio equal-weighted in period 2, you will have to readjust your portfolio holdings. Your investment of \$10,000 is now worth \$12,500, which has to be equally invested in the two assets (\$6,250 in each). So you would need to own $6,250/150 = 41.67$ shares of company 1 and $6250/50 = 125$ shares of company 2. So in order to rebalance your portfolio, you would need to sell company 1 shares and buy company 2 shares.

To keep the value-weighted portfolio value-weighted, you don't need to rebalance your portfolio. This is because the value of your holdings in assets 1 grows proportionately with the increase in the share price of company 1.

3. Evaluating salaries. (12 points, 6 each)

(a) The present value (PV) of the quarterback's contract in millions of dollars is given by

$$\sum_{t=1}^{5} \frac{4}{(1+R)^t}.$$

The present value of the receiver's contract is given by

$$4 + \sum_{t=1}^{5} \frac{3}{(1+R)^t}.$$

If $R = 5\%$, the quarterback's contract has a present value of \$17,317,907, while the receiver's contract has a PV of \$16,988,430. The quarterback signed a better deal.

(b) If $R = 8\%$, the quarterback's contract has a PV of \$15,970,840, while the receiver's contract has a PV of \$15,978,130. In this case the receiver's contract would be better.

As the interest rate increases, the present value of future payments goes down. Both players' contracts are affected by an increase in the interest rates, but the receiver is less affected because he receives more today and less in the future years.

4. Diamonds. (28 points, 7 each)

	Value of your diamonds	Stock index	Treasury	Call option
State 1: Boom	10	14	10	10
State 2: recession	4	8	10	4
Price today		10	9	

(a) Consider a portfolio of purchasing Q_s shares of the stock index and Q_b shares of the Treasury bond. For this portfolio to replicate the payoff of your diamonds in both states of the economy, we must have

$$Q_s \cdot 14 + Q_b \cdot 10 = 10$$
$$Q_s \cdot 8 + Q_b \cdot 10 = 4.$$

Solving this system of equations, we obtain $Q_s = 1$ and $Q_b = -0.4$. By the LOOP, the value of your diamonds today must be the same as the price of this replicating portfolio, which is $Q_s \cdot 10 + Q_b \cdot 9 = 10 - 0.4 \cdot 9 = 6.4$.

(b) The payoff of a call option with strike \$4 is $\max(S-4,0)$, which in this case equals \$10 in a boom and \$4 in a recession. But this is exactly the payoff of your diamonds too. Hence the call option must have the same price as the diamonds, namely \$6.4.

(c) The payoff of the put option in both states of the world is zero. Hence the price must also be zero, otherwise there would be an arbitrage opportunity. (Put-call parity would give you the same answer too.)

(d) The call options pay \$3 in a boom and nothing in a recession. A portfolio that replicates this is to purchase 1/2 share of the stock index and -0.4 shares of the bond (you can solve for this as in part a). The price of this portfolio is $1/2 \cdot 10 - 0.4 \cdot 9 = 5 - 3.6 = 1.4$. Since $1,2 < 1.4$, Downhill is selling the call option at too low a price. The arbitrage trade is to buy a call from Downhill and sell the replicating portfolio, i.e., sell 1/2 share of the stock and buy 0.4 shares of the bond. The payoff of doing this trade today is \$.20, and the payoff tomorrow is zero in both states of the economy.

Downhill is underpricing the calls. It is likely that the demand for Downhill calls is going to go up very steeply. In response to that, Downhill is likely to raise the price of call options.

5. Football. (24 points, 8 each)

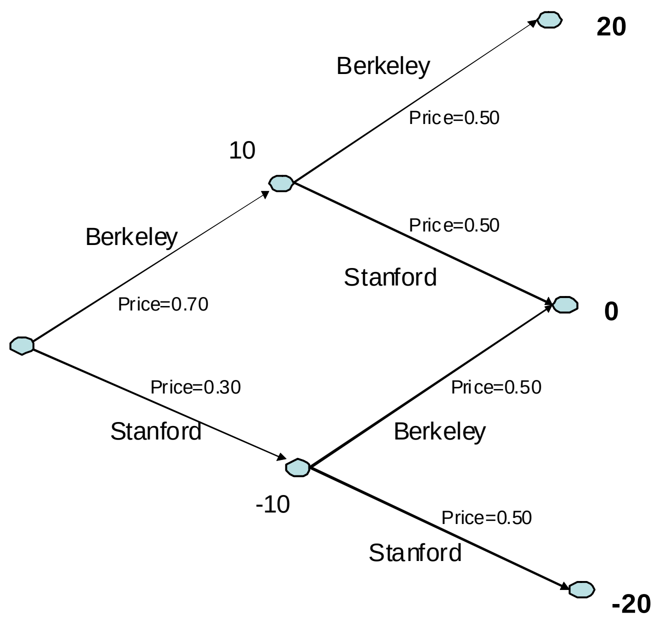
a) See the figure. There are three final states of the world: Berkeley wins by 20 points; it is a tie; Stanford wins by 20 points.

b) (i) To obtain \$1 iff Berkeley wins, do the following: Buy 1/2 share of the Berkeley security at date zero. If Berkeley is ahead at half time, invest your proceeds of \$0.5 in one share of the Berkeley security. If Stanford is ahead at half time, you have nothing left to invest. The price of this strategy at date zero is \$0.35.

(ii) To obtain \$1 iff Stanford wins, buy 1/2 share of the Stanford security at date zero, and re-invest your proceeds of \$0.5 if Stanford is ahead at half time in the Stanford asset. The price of this strategy is \$0.15.

(iii) To obtain \$1 iff it is a tie, buy 1/2 share of both securities at date zero. This gives you \$0.5 at half time irrespective of the outcome. Now, if Stanford is ahead, buy 1 share of Berkeley, and conversely, if Berkeley is ahead buy 1 share of Stanford. The price of this strategy at date zero is \$0.5.

c) Holding fixed the prices of the Stanford and Berkeley securities, the answer does not depend on the probabilities. This is because we did not use the probabilities in deriving the prices of the various AD securities in b). In other words, the probabilities affect the prices of the AD securities in b) only insofar as they change the prices of the Berkeley and Stanford assets.


20
Berkeley
Price=0.50
10
Price=0.50
Berkeley
Stanford
Price=0.70
0
Price=0.30
Price=0.50
Stanford
Berkeley
-10
Price=0.50
Stanford
-20