

## Economics 136. Financial Economics

Sample midterm 1B, Fall 2009, Suggested solutions

### 1. True or false. (20 points, 5 each)

(i) True. The price of a zero-coupon bond is  $P = F/(1 + R)^T$  and when  $R > 0$ , this is less than  $F$ .

(ii) False. A price weighted portfolio means that you hold an equal number of shares of each company in your portfolio, and hence requires no rebalancing when prices change.

(iii) False. By put-call parity,  $C_0 = S_0 + P_0 - X/(1 + R_f)$ . Since  $S_0$ ,  $X$  and  $R_f$  are unchanged, an increase in the put price  $P_0$  must result in an increase in the call price  $C_0$ .

### 2. Mortgage-backed securities(28 points, 7 each)

	Stock index	Treasury	Mortg pool	Tranch A	Tranch B
State 1: good times	16	10	20	10	10
State 2: recession	8	10	8	0	8
Price today	10	9			

(a) To construct AD1, consider a portfolio of  $x$  shares of the stock index and  $y$  shares of the bond. For this to be a replicating portfolio, we need

$$\begin{aligned}16x + 10y &= 1 \\8x + 10y &= 0.\end{aligned}$$

Solving these, we find  $x = 0.125$  and  $y = -0.1$ . Hence by the LOOP, the price of AD1 must be  $10 \cdot (0.125) + 9 \cdot (-0.1) = \$0.35$ . To price AD2, note that the price of a portfolio of AD1 and AD2 must cost \$0.9, because it gives a sure payoff of \$1 in both states, just like 1/10 of a share of the Treasury-bond. As a result, the price of AD2 must be  $0.9 - 0.35 = \$0.55$ . The market is complete, because both AD securities exist.

(b) The payoffs of the mortgage pool can be replicated with a portfolio of 20 shares of AD1 and 8 shares of AD2. Hence the price of the mortgage pool should be  $20 \cdot 0.35 + 8 \cdot 0.55 = 11.4$ .

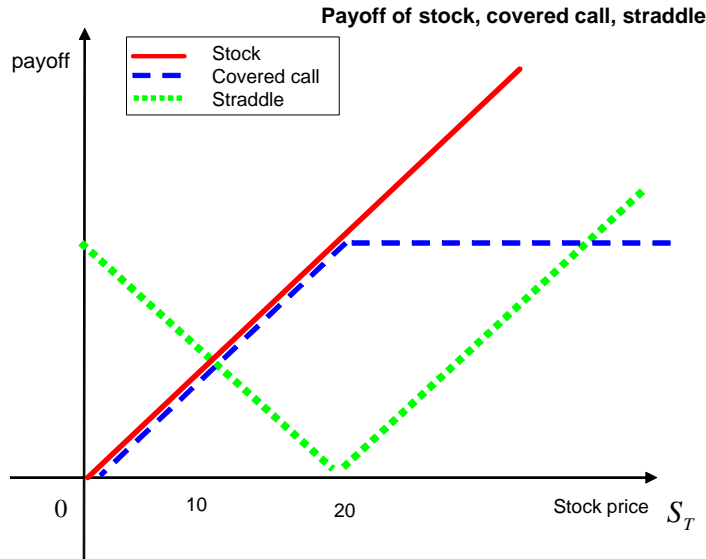
(c) The payoffs of tranches A and B are given in the table.

(d) Since tranche A can be replicated as 10 shares of AD1, its price should be \$3.5. Since Tranch B can be replicated as 10 shares of AD1 and 8 shares of AD2, its price should be 7.9.

### 3. Option portfolios (20 points, 5 each)

a)-c) See the figure.

d) The straddle is the best, because it pays both when the price rises a lot and when it falls a lot.



**4. Bond pricing** (14 points, 7 each)

(a) The semi-annual yield  $R_s$  must satisfy  $(1 + R_s)^2 = 1.1025$  which implies  $R_s = 5\%$ .  
The bond price is then

$$\begin{aligned}
 P &= \sum_{t=1}^{30} \frac{C}{(1 + R_s)^t} + \frac{F}{(1 + R_s)^{30}} = \frac{C}{R_s} \left( 1 - \frac{1}{(1 + R_s)^{30}} \right) + \frac{F}{(1 + R_s)^{30}} \\
 &= 9223.47 + 2313.77 = 11537.25
 \end{aligned}$$

(b) The bond is trading at face value, so the coupon rate and yield must be the same. The semi-annual coupon rate is 6%, thus the semi-annual yield must also be 6%, and then the annual yield is given by  $(1.06)^2 = 1.1236$ , i.e., the annual yield is  $R = 12.36\%$ . Thus the annual yield must have risen by 2.11 percentage points.

**5. Who should buy long term bonds?** (20 points, 4 each)

(a) The total rate of return between  $t=0$  and  $t=2$  if there was a boom is  $1.042 * 1.06 - 1 = 10.5\%$ . If there was a recession, then the total rate of return is  $1.042 * 1.02 - 1 = 6.3\%$ .

(b) The net holding period return on the T-bond is  $F/P_0 - 1 = 100/92 - 1 = 8.7\%$ .

(c) The price of the Treasury bond is the discounted value of its future payment. Since both the T-bond and the T-bill are riskfree between  $t=1$  and  $t=2$ , the appropriate discount rate is the T-bill rate, which is 6% if the economy is in a boom, yielding  $P_1 = 100/1.06 = \$94.34$ . Similarly, if the economy is in a recession, we obtain  $P_1 = 100/1.02 = \$98.04$ .

(d) The return on the Treasury bond between  $t=0$  and  $t=1$  is  $94.34/92 - 1 = 2.5\%$  if the economy enters a boom, and  $98.04/92 = 6.6\%$  if the economy enters a recession.

(e) The T-bill is safe for a short-term investor who invests between  $t = 0$  and  $t = 1$  only. The T-bill is risky if you invest between  $t = 0$  and  $t = 2$ , i.e., for a long-term investor. The safe asset for a long-term investor is the T-bond.