Final Exam

Ec 234A, Fall 2009

Problem 1.

Answer the following short questions in no more than two sentences.

(i) In the standard Lucas tree model, what happens to the variance of consumption growth and wealth growth as a function of the time horizon? Illustrate with a figure.

(ii) In an economy where CAPM holds, would anybody hold a stock that has a lower Sharpe-ratio than the market portfolio? Explain.

(iii) In the standard Lucas tree economy, does the riskfree rate increase or decrease when expected consumption growth increases? Explain.

(iv) What is the reversal effect? How is it related to the value effect?

(v) People sometimes say that the Fama-French model explains the value and size effects by design. What logic does this statement refer to?

Problem 2.

Consider an economy over three dates, $t = 0$, $t = 1$ and $t = 2$, where the riskfree rate is zero, and the risky asset has a terminal payoff $V = F + \varepsilon$ at $t = 2$. Assume that F and ε are independent, normally distributed, both with mean zero and unit standard deviation. The supply of the risky asset is zero. Assume that the price of the risky asset at $t = 0$ is $P_0 = 0$.

(a) A unit mass of news-watchers observe F (but not ε) at $t = 1$, and form biased expectations $E^n[V] = (1 - \delta) F$, where $0 \leq \delta \leq 1$ measures their bias ($\delta = 0$ means no bias). They correctly perceive variances, and invest at $t = 1$ to maximize

$$\mathrm{E}^n[w] - (1/2)\,\mathrm{Var}(w)$$

where w is their terminal wealth. Show that news-watchers demand $D^n = F(1 - \delta) - P_1$ shares at $t = 1$. What is the equilibrium price P_1 in an economy with only news-watchers? Does it equal fundamental value F? Why?

(b) Now suppose that rational (unbiased) arbitrageurs also participate at $t = 1$, have the same preferences as news-watchers, and also observe F. What is their demand at $t = 1$? Do arbitrageurs act as "value investors" (buy when the price is low), or "momentum investors" (buy when the price has increased)? If the mass of arbitrageurs is m, what is the equilibrium price? Why doesn't it equal fundamental value? What happens to the extent of mispricing $(P_1 - F)/F$ when m increases?

(c) Now suppose that arbitrageurs do not observe F, but can condition their demand on the price P_1. To solve for equilibrium, guess that arbitrageurs completely learn F from

P_1. Given their knowledge of F, how many shares do they demand? What is the implied equilibrium price? Is it different from (c)? Verify your guess: does this price fully reflect F?

(d) Continuing, given P_1, how do arbitrageurs compute F? Use this to express the total demand of arbitrageurs as $D^a = m \cdot \phi P_1$ with some constant ϕ. What is the sign of ϕ? Do arbitrageurs act as "value investors" or "momentum investors"? Why?

(e) Now suppose that the mass of arbitrageurs is $m = \theta N$ where N is known but θ is uncertain, due to fluctuations in arbitrage capital. Do you expect that arbitrageurs will fully learn F now? Assuming that they still buy ϕP_1 shares with a ϕ that has the same sign as above, describe the signal extraction problem they face when observing P_1. Relate this to the externality that momentum traders impose on each other in the Hong and Stein (1999) model. Suppose that $N \to \infty$ (but θ continues to be random). Do you expect that prices will converge to fundamentals? Does higher competition by arbitrageurs always make the market more efficient?

Problem 3.

Consider a two-period endowment economy with a unit mass of consumers. Each agent has log utility and consumes only at $t = 2$. Agents trade in financial markets at $t = 1$, but all transactions are settled at $t = 2$ (i.e., the price of an investment is paid at $t = 2$). At $t = 2$, there are two states of the world: with probability 1/2, there is a boom and aggregate endowment equals m; with remaining probability 1/2 there is a recession and the aggregate endowment is $(1 - a)\, m$, where $0 < a < 1$.

a) Suppose that all consumers' endowments are the same and equal the aggregate endowment. At $t = 1$ a single financial asset is traded, which is a claim to the aggregate endowment, and has price p (i.e., a fix payment p must be made at $t = 2$ in both states for each share purchased). If an agent purchases s shares, how much does he consume in the two states of the world? (Your answer should account for his endowment, the payoff of his investment, and the price of the investment). What is his maximization problem given p? What is the first order condition for the optimal choice of s?

b) The net supply of the risky asset is zero. Using that all agents are identical, substitute this into the first order condition from (a). What is the equilibrium price p? What happens to p as $a \to 1$? Why?

c) Now suppose that at $t = 2$, in the good state all agents have endowment m, but in the bad state, a random share $(1 - b)$ of agents get m, while the remaining share b of agents get $(1 - a/b)\, m$, where $a < b \leq 1$. The former group is lucky, the latter is unlucky ex post; but consumers are still identical ex ante. Show that the aggregate endowment in the two states is the same as before. If the only risky asset is a claim to the aggregate endowment, how much does an agent who purchases s shares consume in each eventuality? What is the agent's first order condition?

d) Using that agents are ex ante identical, solve for the equilibrium price p. Compute the special cases when $b = 1$ (no idiosyncratic risk) and when $b \to a$ (high idiosyncratic risk). Based on these results, do you think p is increasing or decreasing in b? Explain the intuition. (You can also compute dp/db and check the sign.) Does b affect aggregate consumption?

e) Now suppose that insurance markets open at $t = 1$ allowing investors to hedge idiosyncratic risk. What will be the price of the risky asset now? How does it depend on b? Relate your results in (b), (d) and (e) to empirical tests of the consumption-based asset pricing model based on aggregate consumption.

Problem 4.

A key role of financial markets is information aggregation.

a) Explain the difference between competitive equilibrium and rational expectations equilibrium.

b) Describe the conceptual problem with fully efficient markets where prices reflect all available information. How do Grossman and Stiglitz resolve this problem?

c) Does price always reflect fundamental value when some agents have mistaken beliefs? Will biased agents be necessarily driven out of the market? Why?

d) Describe an agency problem between arbitrageurs and financiers, and explain how it might limit arbitrage and prevent prices from converging to fundamentals.

e) [Extra credit] What is your opinion of this literature? Which directions seem promising to you?