Final Exam

Ec 234A, Fall 2009

Suggested solutions

Problem 2.

(a) Purchasing D^n shares results in terminal wealth $w = w_0 + D^n (V - P_1)$, which has biased expectation $w_0 + D^n ((1-\delta) F - P_1)$ and variance D^{n2}. Hence the investor maximizes $D^n ((1-\delta) F - P_1) - (1/2) D^{n2}$ which leads to $D^n = F(1-\delta) - P_1$. In equilibrium this demand equals the zero supply, so that $P_1 = F(1-\delta)$. Prices depart from fundamentals because traders have biased expectations.

(b) Arbitrageurs solve the same problem with $\delta = 0$, hence each of them demands $F - P_1$. This is decreasing in P_1: arbitrageurs act as value investors who buy more when prices are lower. Market clearing requires $m(F - P_1) + F(1-\delta) - P_1 = 0$, which implies $P_1 = F - F \cdot \delta/(m+1)$. Price still departs from F because arbitrageurs have finite positions due to risk aversion. $(P_1 - F)/F = -\delta/(m+1)$ goes to zero as $m \to \infty$: when arbitrage capital increases, mispricing goes down.

(c) If they completely learn F, then their demand is $F - P_1$ as before; hence the equilibrium price is $P_1 = F - F \cdot \delta/(m+1)$ as before. This is the same as (c), and it is a constant multiple of F, hence the guess is verified.

(d) Arbitragurs can compute $F = P_1/(1 - \delta/(m+1))$. Hence their total demand is $m \cdot (F - P_1) = m \cdot P_1 \cdot [1/(1-\delta/(m+1)) - 1] = m \cdot P_1 \cdot \delta/(m+1-\delta)$. We have $\phi = \delta/(m+1-\delta) > 0$, hence arbitraguers now act as momentum investors. Intuitively, a high price must reflect good news, but given that biased news-watchers do not fully incorporate this effect, a momentum strategy is optimal.

(e) Arbitrageurs won't learn F from P_1, because P_1 will be contaminated by noise in the supply of arb capital m. Their signal extraction problem is that when P_1 is high, it may be due to high fundamental value, in which case momentum is optimal, or a high volume of momentum trading, in which case the asset may be overpriced. This is like the Hong and Stein externality: previous momentum traders may bid up the price too much, reducing the profitability of other momentum investors. Prices won't converge to fundamentals even with many arbitrageurs, because the risk in the amount of arbitrage capital is also scaled up, preventing them from figuring out the fundamental value. Compeition need not improve market efficiency, although it does drive down arbitrageurs profits. See also "Sophisticated Investors and Market Efficiency" by Jeremy Stein, Journal of Finance, 2009.

Problem 3.

(a) One unit of the claim pays m in the good state and $(1-a)m$ in the bad state, and has a price p to be paid in both states. Thus consumption in the good state is $m + s(m - p)$, and consumption in the bad state is $m(1-a)+s(m(1-a)-p)$. Each consumer maximizes

$$\max_s \frac{1}{2}\log\left(m+s(m-p)\right)+\frac{1}{2}\log\left(m(1-a)+s(m(1-a)-p)\right)$$

which has the FOC

$$\frac{1}{2}\frac{m-p}{m+s(m-p)}+\frac{1}{2}\frac{m(1-a)-p}{m(1-a)+s(m(1-a)-p)}=0.$$

(b) Since $s=0$ we get

$$\frac{m-p}{m}+\frac{(1-a)m-p}{(1-a)m}=0$$

and hence

$$p=\frac{2m(1-a)}{2-a}.$$

When $a\to 1$ this goes to zero, because the bad state becomes increasingly painful.

(c) In the good state, the endowments are unchanged; in the bad state, aggregate endowment is $(1-b)\,m+b\,(1-a/b)\,m=(1-a)\,m$ as claimed. The maximization problem is now

$$\max_s \frac{1}{2}\log\left(m+s(m-p)\right)+$$
$$+\frac{1}{2}\left[(1-b)\log(m+s(m(1-a)-p))+b\log\left(m(1-a/b)+s(m(1-a)-p)\right)\right]$$

and has the first order condition

$$\frac{m-p}{m+s(m-p)}+(1-b)\frac{m(1-a)-p}{m+s(m(1-a)-p}+b\frac{m(1-a)-p}{m(1-a/b)+s(m(1-a)-p}=0.$$

d) Using that agents are ex ante identical, solve for the equilibrium price p. Compute the expected return as in (b). Is it higher or lower? How does it depend on b? Explain the intuition.

(d) Substituting $s=0$ yields

$$0=\frac{m-p}{m}+\frac{(1-b)((1-a)m-p)}{m}+\frac{b((1-a)m-p)}{(1-a/b)m}$$

which implies

$$p=m\frac{1+(1-a)(1+ab/(b-a))}{2+ab/(b-a)}.$$

When $b=1$ this becomes the answer to (b), i.e., $2m(1-a)/\left(2-a\right)$. When $b\to a$, this becomes $m\,(1-a)$, which is smaller. Thus when b is lower (high idiosyncratic risk) the price is lower. Intuitively, expected pain in the recession is higher when b is low, because of higher idyosincratic risk in bad times. Aggregate consumption is not affected.

(e) With insurance markets, we get back the same prices as in (b), because consumers can completely share the risk of being unlucky in the recession. The lesson is that with idiosyncratic risk and incomplete markets, aggregatge consumption cannot be used for asset pricing.