Ec 234A, Macroeconomic Finance

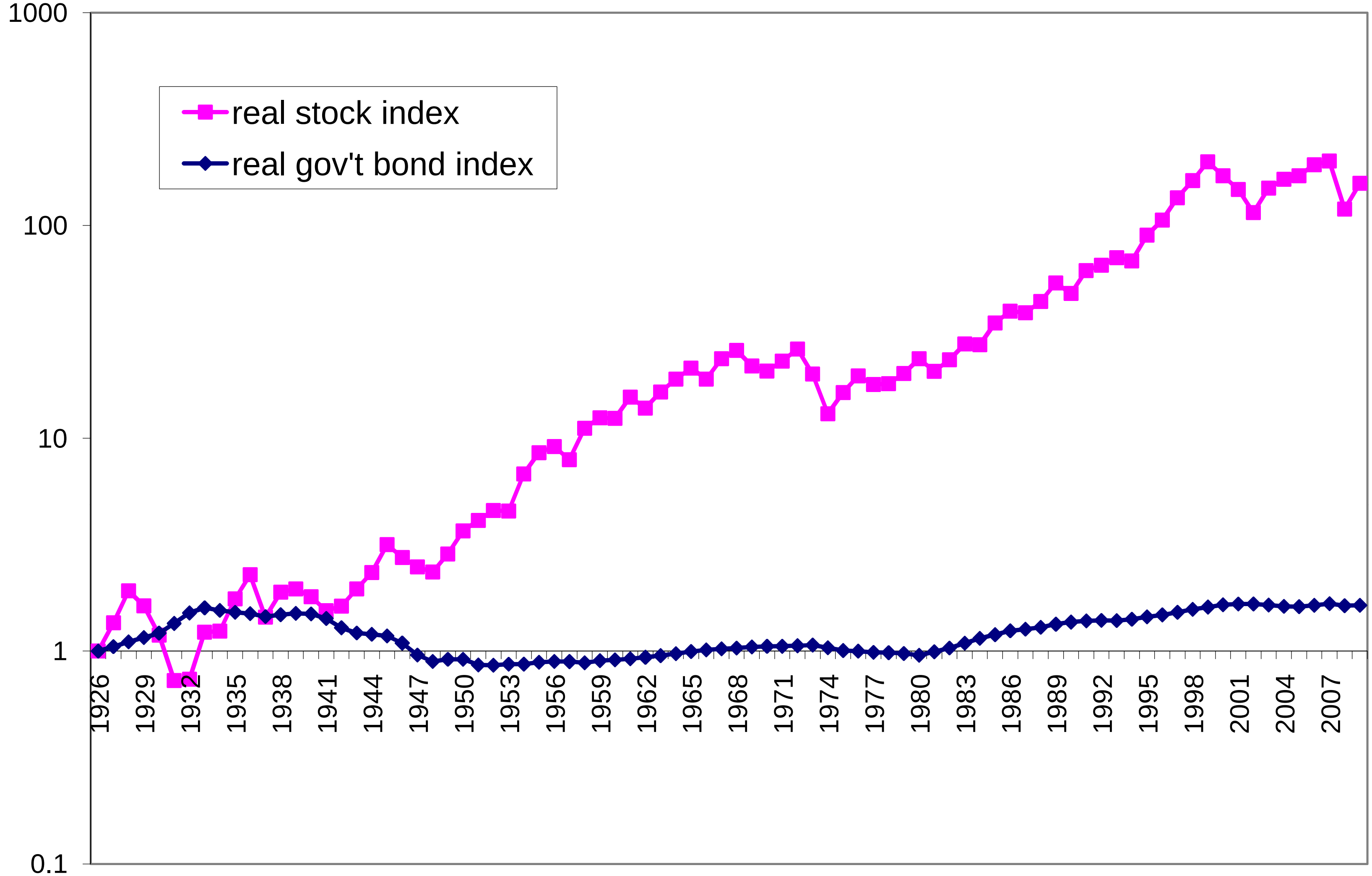
Lecture 1

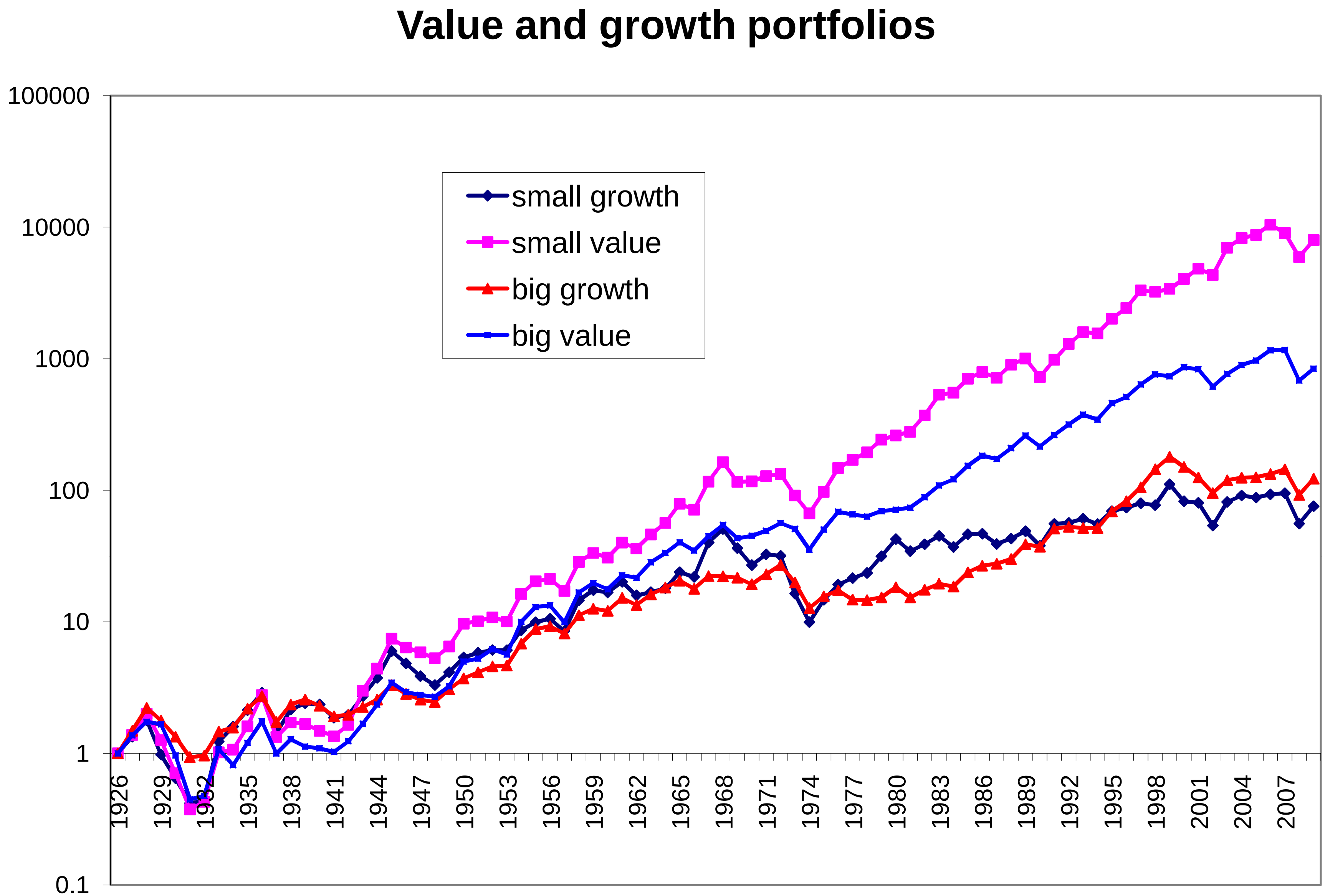
January 18, 2011

This course focuses on questions like

- How much do stocks earn relative to bonds, and why?
- Why do some stocks earn higher returns than others?
- Why are stock prices volatile, when dividends are smooth?
- What predicts stock returns, and why?
- How should and how do households invest their financial wealth?
- What causes bubbles and crashes?
- What are the limits to arbitrage?

Real stock and bond performance, log scale





Value and growth portfolios
100000
10000
1000
100
10
1
0.1
small growth
small value
big growth
big value
1926
1929
1932
1935
1938
1941
1944
1947
1950
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2004
2007

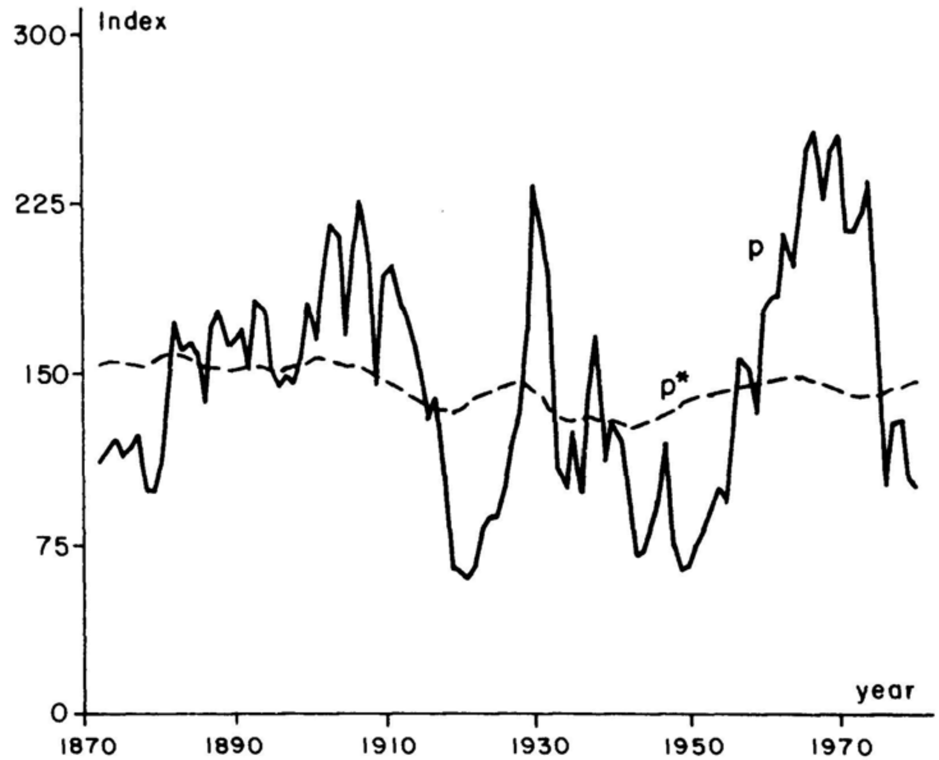


FIGURE 1

Note: Real Standard and Poor's Composite Stock Price Index (solid line p) and *ex post* rational price (dotted line p^*), 1871–1979, both detrended by dividing a long-run exponential growth factor. The variable p^* is the present value of actual subsequent real detrended dividends, subject to an assumption about the present value in 1979 of dividends thereafter. Data are from Data Set 1, Appendix.

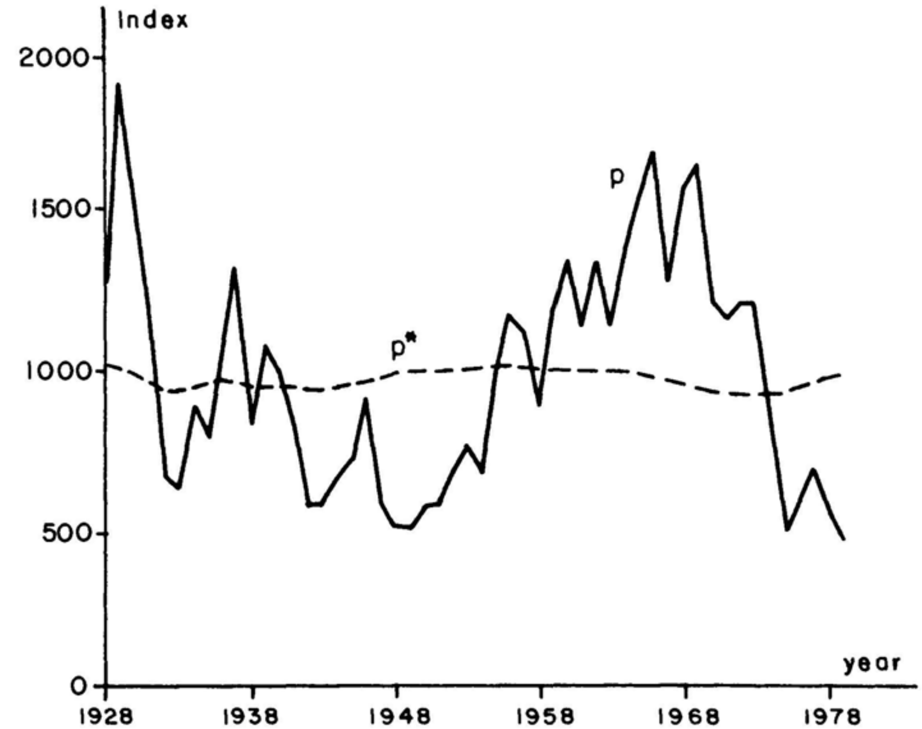
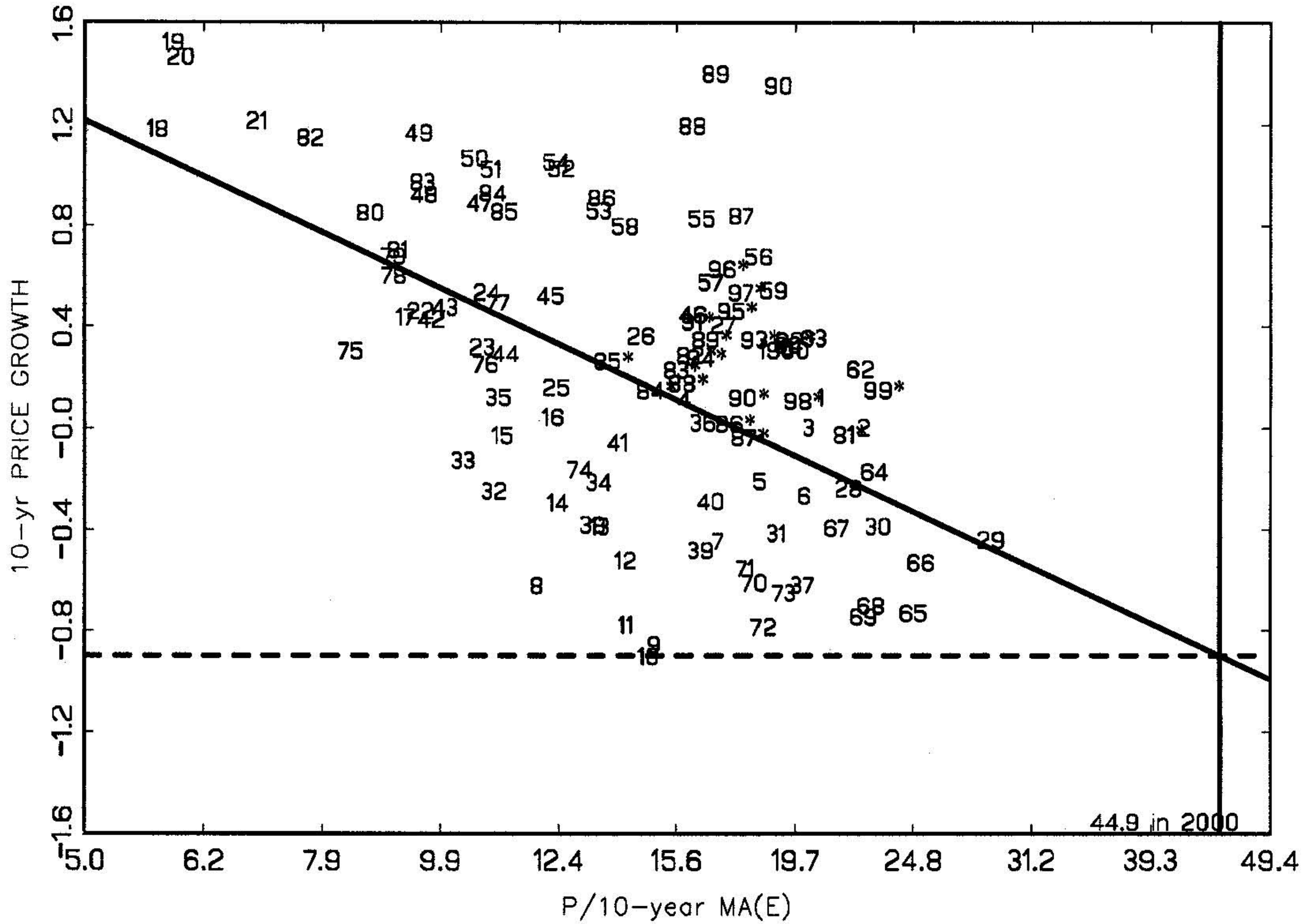


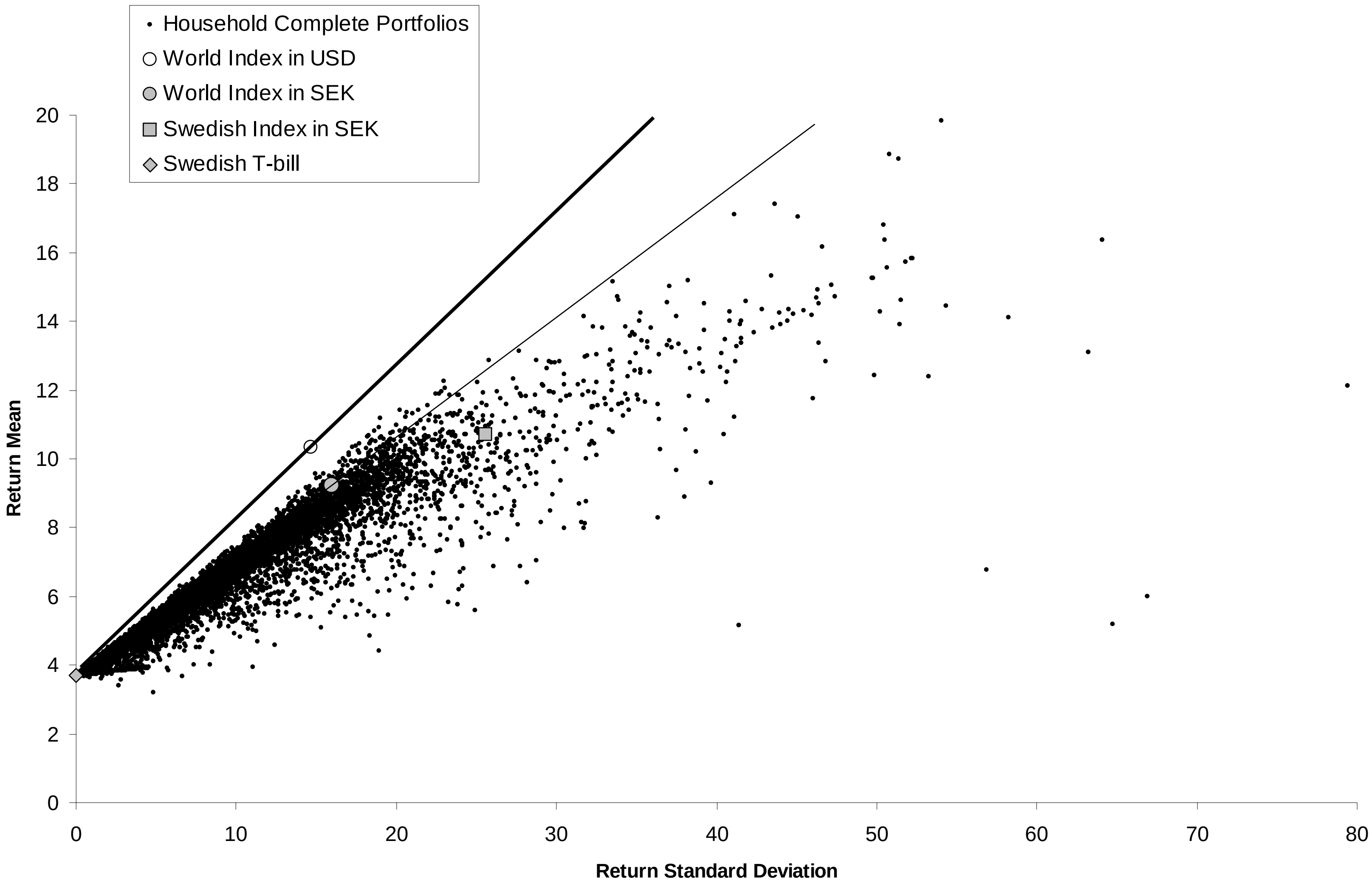
FIGURE 2

Note: Real modified Dow Jones Industrial Average (solid line p) and *ex post* rational price (dotted line p^*), 1928-1979, both detrended by dividing by a long-run exponential growth factor. The variable p^* is the present value of actual subsequent real detrended dividends, subject to an assumption about the present value in 1979 of dividends thereafter. Data are from Data Set 2, Appendix.

10-year PRICE GROWTH vs P/10-year MA(E)



b. Complete Portfolios



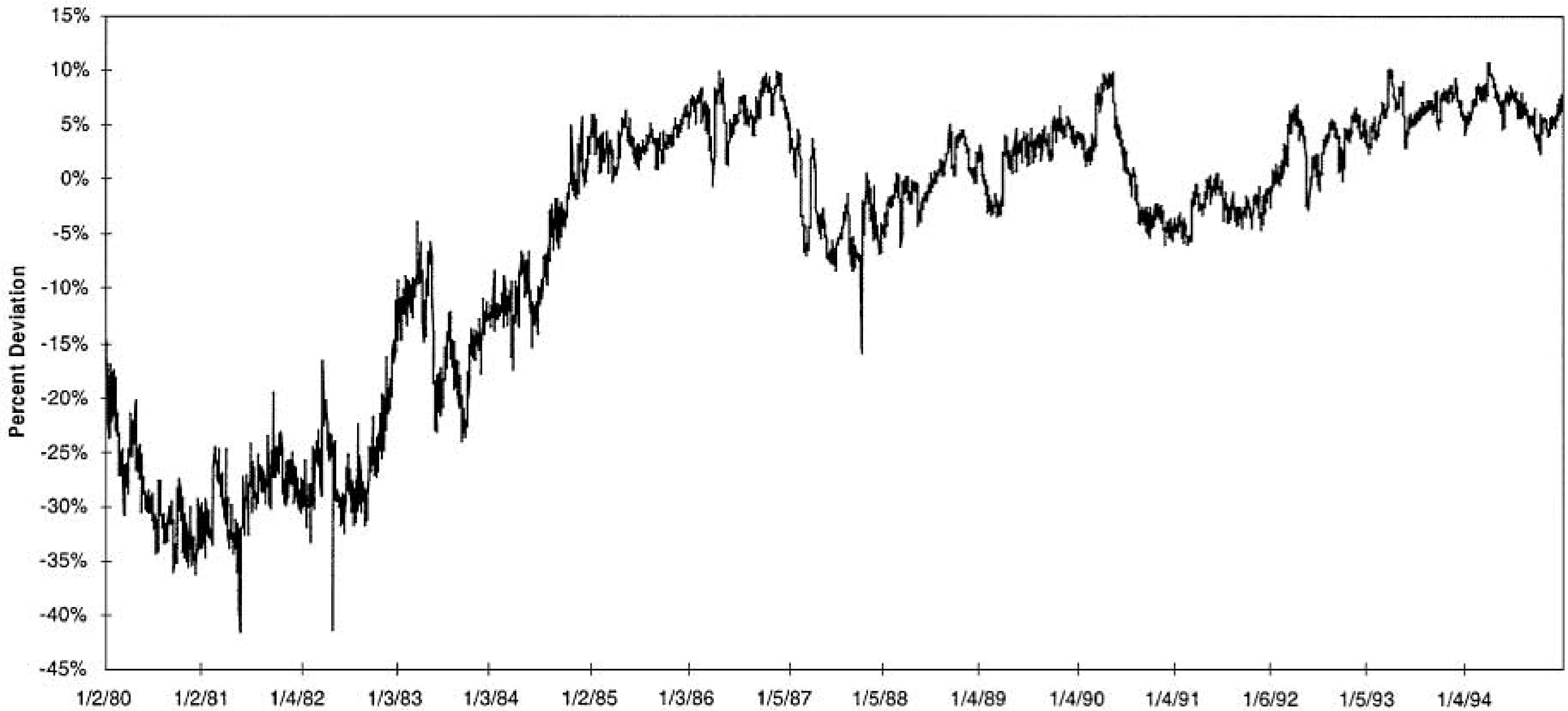


Fig. 1. Log deviations from Royal Dutch/Shell parity. Note: This figure shows on a percentage basis the deviations from theoretical parity of Royal Dutch and Shell shares and ADRs traded on the NYSE. Data are from the Center for Research in Security Pricing (CRSP).

Outline for Today

1. Risk aversion
2. Utility functions
3. Principles of portfolio choice

1. Risk Aversion

- Much of finance uses expected utility framework: $\max \mathsf{E} u\left(w\right)$

- Agent is *risk averse* if he dislikes all zero-mean risk:

$$\mathsf{E}u(\tilde{z}) \leq u(\mathsf{E}\tilde{z}).$$

- *Jensen's inequality.* $\mathsf{E}f(\tilde{z}) \leq f(\mathsf{E}\tilde{z})$ holds for all random variables $\tilde{z}$ iff $f(.)$ is concave.
  - Natural measure of risk aversion: u''
  - Need to scale to make it unit-free

- *Absolute risk aversion* is $A = -u''/u'$.

Arrow-Pratt approximation

- Let $\widetilde{y} = k\widetilde{x}$ a pure risk, i.e., $\mathsf{E}\widetilde{x} = 0$, where $k \geq 0$ is scales risk. Write risk premium as $g(k)$:

$$Eu(w_0 + k\widetilde{x}) = u(w_0 - g(k))$$

  so $g(0) =$

- Differentiate in k:

$$\mathsf{E}\widetilde{x}u'(w_0 + k\widetilde{x}) = -g'(k)u'(w_0 - g(k))$$

  so $g'(0) =$

- Differentiate in k again:

  $$\mathsf{E}\widetilde{x}^2 u''(w_0 + k\widetilde{x}) =$$
  $$= g'(k)^2 u''(w_0 - g(k)) - g''(k)u'(w_0 - g(k))$$

  so that

  $$g''(0) = \frac{-u''(w_0)}{u'(w_0)}\mathsf{E}\widetilde{x}^2 = A(w_0) \cdot \mathsf{E}\widetilde{x}^2.$$

- Taylor-approximation of $g(k)$ around zero

  $$g(k) \cong g(0) + kg'(0) + \frac{1}{2}k^2 g''(0) + O(k^3)$$

- Hence risk premium is

  $$\pi \cong \frac{1}{2}k^2\mathsf{E}\widetilde{x}^2 \cdot A(w_0) = \frac{1}{2}A(w_0) \cdot \mathsf{E}\widetilde{y}^2$$

- Tolerance to additive risk depends on $A(w_0)$.

- *Second-order risk aversion:* Risk premium is proportional to variance.

  - People are approximately risk neutral to small risks.

  - E.g., if π for fair gamble of $\pm$\$1000 is \$50, then π for $\pm$\$100 gamble is 50 cents.

- Rabin critique: in practice, people are relatively more averse to small than to large risks.

Relative Risk Aversion

- Multiplicative risk

$$\widetilde{w} = w_0(1 + k\widetilde{x})$$

- Define $\widehat{\pi}$ as share of wealth one would pay to avoid risk, then

$$\widehat{\pi} \cong \frac{1}{2}k^2 \mathsf{E}\widetilde{x}^2 \cdot w_0 A(w_0) = \frac{1}{2} R(w_0) \cdot \mathsf{E}\widetilde{y}^2$$

  where $R(w_0)$ is the *coefficient of relative risk aversion*

$$R(w_0) = \frac{-u''(w_0)}{u'(w_0)} w_0.$$

- Tolerance to multiplicative risk depends on relative risk aversion $R(w_0)$.

2. Utility functions

- A common class of utility functions in finance is "hyperbolic absolute risk aversion" (HARA)

$$u(z) = \varsigma \cdot (\eta + \frac{z}{\gamma})^{1-\gamma}$$

- Risk aversion for this class:

$$\begin{aligned} A(z) &= \left(\eta + \frac{z}{\gamma}\right)^{-1} \\ R(z) &= z\left(\eta + \frac{z}{\gamma}\right)^{-1} . \end{aligned}$$

Special cases

1. *Quadratic utility*. Implies increasing absolute risk aversion. Has a bliss point where $u' = 0$. Tractable with additive risk.

2. *Exponential utility*, (CARA=constant absolute risk aversion). To obtain constant coefficient A, need

   $$-u''(z) = Au'(z)$$

   solving ODE gives

   $$u(z) = \frac{-\exp(-Az)}{A}$$

   Tractable with additive, normally distributed risk. Increasing relative risk aversion; requires same dollar compensation for an additive risk at all wealth levels.

3. *Power utility* (CRRA, constant relative risk aversion). $\eta = 0$, $\gamma > 0$. We have $R = \gamma$. For $\gamma \neq 1$

$$u(z) = \frac{z^{1-\gamma}}{1-\gamma}$$

for $\gamma = 1$

$$u(z) = \log(z).$$

Tractable with multiplicative, lognormal risk. Requires same proportional compensation (e.g., return) for proportional risk across all wealth levels. Appealing because implies stationary risk premium and constant interest rates in the presence of economic growth.