

Problem Set 4

Ec 234A, Spring 2011

This problem set is due on April 26.

Problem 1.

Consider a financial market with three types of agents: (i) noise traders; (ii) an insider; (iii) market makers. The market is open for one period, and one risky financial asset is traded. Denote the terminal value of the asset by v , a normally distributed random variable with expected value p_0 and variance σ_v^2 .

The market operates the following way. The risk neutral insider, who has zero endowment of the risky asset, observes v and then places a market order x . Risk neutral market makers observe the total order flow $x + u$, where u is the exogenous demand of noise traders, which is independent of v and normally distributed with mean zero and variance σ_u^2 . Competition among market makers is assumed to result in the market price

$$p = E[v|x + u].$$

The insider behaves strategically, i.e., in deciding on her optimal strategy, she takes into account the effect of her demand x on the price p .

(a) Assume that the market price is a linear function of the total order flow

$$p = \mu + \lambda(x + u).$$

Denote the profit of the insider by $\pi = x \cdot (v - p)$. Compute the expected profit of the insider conditional on v . Solve for the optimal demand of the insider x in terms of λ and μ and v . Express x in the form of $x = \alpha + \beta v$ where α and β are constants. Why is the insider's demand x finite even though he is risk neutral?

(b) Denote the total order flow by $y = x + u$. Note that the price is determined as

$$p = E[v|y] = \mu + \lambda y$$

by assumption. You can view the conditional expectation as a regression of v on a constant and y (by normality, the conditional expectation is a linear function). Express μ and λ using the formulas for the regression coefficients as functions of α and β . Now you have four equations relating α , β , μ and λ . Express all four of these variables using exogenous parameters of the model. Interpret λ as a measure of market illiquidity: when λ is high, prices are very responsive to changes in demand. Explain the comparative statics of λ with respect to σ_v and σ_u .

(c) Compute the unconditional expected profit of the insider. When is it positive? Who loses money in this financial market?

(d) Compute the conditional variance of the price given the information of the insider, $\text{var}[p|v]$. How is it related to the variance of fundamentals σ_v^2 ? Does it depend on the activity of noise traders σ_u^2 ? Explain.