Consumption Commitments: A Foundation for Reference-Dependent Preferences and Habit Formation*

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Abstract

We build a theory of reference-dependent preferences based on adjustment costs in consumption. The main contribution of the theory is that it endogenizes the evolution of the reference point. The reference point generated by our model exhibits several features exogenously assumed in existing theories: it (1) reflects recent expectations, (2) depends on past consumption levels, and (3) diminishes in importance when agents experience large shocks. When the ratio of idiosyncratic to aggregate risk is large, the model is approximately equivalent to standard habit formation specifications in which the reference point is a weighted average of past consumption. We illustrate the implications of endogenizing the reference point using three applications: aggregate consumption dynamics, changes in policy parameters, and the welfare cost of shocks. In each application, our model of endogenous reference points confirms certain intuitions from existing theories but yields some starkly different predictions. For example, reducing idiosyncratic risk can *raise* the welfare cost of aggregate shocks by making reference points more persistent.

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1 Introduction

Models of reference dependent preferences – in which utility depends upon the difference between outcomes and a pre-determined benchmark – are now widely used in economics. A central question in these models is how the reference point is determined, to which the existing literature has proposed two general answers. In habit formation and status-quo models, the reference point is modeled as an average of recent consumption. In these models, the agent evaluates outcomes relative to recent experience, to which he has habituated himself.[1] In contrast, a more recent literature has argued that the reference point is determined by recent expectations, i.e. the agent compares outcomes to what he expects to obtain.[2] The difference between these two competing approaches is analogous to the debate between adaptive- and rational-expectations theories in macroeconomics.

Although existing theories specify the determinants of the reference point, they do not endogenize its *evolution*, i.e., the dates when it is adjusted and the speed with which it responds to shocks. The dynamics of the reference point are either determined by exogenous weights on past consumption (as in habit models such as Constantinides, 1990) or by past expectations at an exogenous time lag (as in Koszegi and Rabin 2009). In this paper, we propose a theory of reference dependence and habit formation that endogenizes the evolution of reference points. Our model confirms many intuitions from existing reference-dependent models but also yields some starkly different predictions because of endogenous changes in the reference point. In this sense, our analysis underscores the benefits of providing explicit microfoundations for reduced form models (Lucas 1976, Stigler and Becker 1977, Postlewaite 1998).

The central premise of our theory of reference dependence is that changing the consumption of certain goods – which we term "consumption commitments" – is costly. These costs could reflect either transaction costs or mental costs such as the effort required in changing plans (Grossman and Laroque 1990, Chetty and Szeidl 2007). Consumption commitments act as a reference point by altering the agent's indirect utility over total consumption. Because commitments are themselves chosen to maximize expected utility, we obtain an explicitly dynamic theory for the evolution of

[1] Ryder and Heal (1973) specify habit as an exponential average of past consumption. Sundaresan (1989), Constantinides (1990), Campbell and Cochrane (1999) and Boldrin, Christiano and Fisher (2001) use variants of this model in macro-finance. In macroeconomics, Carroll, Overland and Weil (2000), Fuhrer (2000), Christiano, Eichenbaum, and Evans (2003) and a large literature in monetary policy building on this work use models where habit is a function of lagged consumption.

[2] Koszegi and Rabin (2006) develop such a forward looking model. This framework has been applied by Koszegi and Rabin (2007, 2009), Koszegi and Heidhues (2008), Yogo (2008), Crawford and Meng (2009) and others.

the reference point. The only additional primitive that must be specified exogenously to calculate these dynamics is the adjustment cost, a parameter that can at least in principle be measured empirically.

We show that the commitment-based model of reference points unifies the intuitions of many existing models of reference dependence. Our framework is forward-looking: the choice of consumption commitments is based upon expectations about future income. But due to infrequent adjustment, current commitments reflect *recent* expectations, which in turn are also reflected in recent consumption choices by the permanent income hypothesis. As a result, the current reference point is also related to recent consumption, as in backward-looking models. In addition, models of reference dependence sometimes posit that the utility has two components – one that features reference dependence and another that has a neoclassical form – so that the importance of the reference point diminishes for large fluctuations. Our model endogenously generates this property because agents abandon commitments when they face large shocks and thus no longer have reference-dependent preferences. The degree to which these various properties are manifested depends upon the primitives of the environment and the realization of shocks, creating rich dynamics for the reference point.

We characterize these dynamics by analyzing aggregate behavior in a model economy populated by many agents. Individual discrete adjustments in the reference point are smoothed out in the large population, allowing us to focus on the central trends. This environment allows us to establish a tight connection between our forward-looking model and the most popular backward-looking model of the reference point, habit formation, which is typically used to study macroeconomic aggregates. Our main theoretical result is that when the ratio of idiosyncratic to aggregate consumption risk is large, aggregate commitments are well approximated by a weighted average of past consumption with fixed weights.[3] Hence, aggregate dynamics are close to a representative agent habit-formation model where current habit is an average of past consumption. To understand this result, note that the impulse response to aggregate shocks in our model depends upon the distribution of agents in the inaction region for commitment consumption. Aggregate shocks perturb this distribution, while idiosyncratic shocks push it back towards its steady state. When idiosyncratic risk is large, the second effect dominates, and hence on most dates the distribution remains close to its steady

[3] Our characterization is analytical. Previous studies of aggregate dynamics with adjustment costs use numerical techniques (Marshall and Parekh, 1999), time-dependent adjustment (Lynch, 1996, Gabaix and Laibson, 2001, Reis, 2006) or approximate solutions (Caballero and Engel, 1993). These studies focus on a model with a single illiquid good, as in Grossman and Laroque (1990).

state, creating dynamics that are well approximated by a fixed, state-independent impulse response. This in turn can be generated with a habit model that has fixed weights. Hence, backward-looking habit models can be viewed as a "reduced form" representation of forward-looking reference point models based on adjustment costs. From an applied perspective, this equivalence result shows that reduced-form habit models can provide a convenient description of behavior in the empirically relevant case where idiosyncratic risk is large relative to aggregate risk. If the fundamental source of habit is physical or mental adjustment costs, one can calibrate the habit weights by specifying these adjustment costs at the microeconomic level.

We illustrate the implications of endogenizing the reference point using three applications. We first consider the model's implications for aggregate consumption behavior. Two well-documented empirical regularities are that consumption does not respond fully to contemporaneous shocks (excess smoothness, Deaton 1987) and that anticipated changes affect current consumption (excess sensitivity, Flavin 1981). Fuhrer (2000) argues that both of these features of the data can be explained by a habit formation model because habit responds sluggishly to shocks. As shown by our equivalence result, the commitment model also produces sluggish responses in most periods, and therefore also explains excess sensitivity and smoothness. However, such sluggish responses only occur in normal times with moderate shocks. During extreme events such as large recessions and crises, many households will choose to pay the fixed cost of adjustment, and therefore aggregate consumption responds quickly as the reference point becomes less important. Hence our model predicts a breakdown in correlations between macro aggregates and their lags in extreme events due to rapid adjustment. The same mechanism may help explain the evidence for correlation breakdowns in financial markets during extreme events (Longin and Solnik 2001).[4]

Our second application considers the welfare cost of shocks. As in habit models, the reference point created by commitments amplifies the welfare cost of shocks. However, the endogenous evolution of the reference point again modifies the model's predictions for large shocks. Agents can abandon commitments in extreme events, while they are prohibited from doing so in reduced-form models of reference dependence. The welfare cost of large shocks is therefore higher in the habit model than with commitments. This distinction is strongest when the habit stock is slow-moving, in which case the high habit set by past consumption reduces the welfare of the agent for an extended period during an negative shock. For example, in a commitment economy with low idiosyncratic

[4] The inattention model of Reis (2006) also generates excess sensitivity and smoothness of consumption, but does not predict correlations breraking down in extreme events because it features time-dependent adjustment.

risk, consumption is quite sluggish due to infrequent adjustment, implying that the habit model that matches these data would be quite persistent. In this economy, a researcher estimating a standard habit model would incorrectly predict high aversion to extreme events, whereas in the true model agents would simply abandon commitments in response to a large shock.

These two applications show that our model generates different predictions from existing theories because the reference point responds endogenously to *shocks*. In our third application, we show that the endogenous response of the reference point to *primitives* also leads to different predictions. Because commitments are chosen by the consumer, they respond endogenously to policy changes. Hence, the parameters of a reduced-form habit model chosen to match data generated by the commitment economy will change when policy or environmental parameters change. Reductions in risk or in expected growth increase sluggishness in the commitment economy, because agents update commitments less frequently. Reducing idiosyncratic risk – e.g. by expanding social insurance programs – increases welfare, as in models of habit. However, it can also raise the welfare cost of aggregate shocks by making habit more persistent, as individuals change their plans and commitments less frequently. One potential implication is that recessions will last longer in European welfare states, which have large social safety nets, than in economies like the U.S. where idiosyncratic risk is higher.[5] Another implication is that consumption will respond more rapidly to shocks – and recessions may be shorter lived – in rapidly growing economies, where agents change reference points quickly.

In addition to the literature on reference-dependence and habit that motivates this paper, our analysis relates to and builds on several other strands of research. The qualitative similarity between the commitment and reference-dependent models has been pointed out in several recent papers. Dybvig (1995) examines ratcheting consumption demand in a model with extreme habit persistence, and motivates these preferences by pre-commitment in consumption. Flavin and Nakagawa (2008), analyze asset pricing in a two-good adjustment cost model, and note the similarity to habit. Postlewaite, Samuelson and Silverman (2008), Fratantoni (2001), and Li (2003) also study two-good models and note this similarity in different contexts. We contribute to this literature by analyzing aggregate dynamics, presenting formal conditions under which commitments, reference-dependence, and habit formation are similar, and contrasting the implications of exogenous vs. endogenous reference points. Our results call for caution in using habit models that do not

[5] In a general equilibrium model, these effects are likely to be amplified further, when non-adjustment by some agents affects the choices of others.

account for endogenous responses of the reference point.

Our analysis is also closely related to a growing literature that examines the implications of consumption commitments. Postlewaite, Samuelson and Silverman (2008) show that commitments generate incentives to bunch uninsured risks together, potentially explaining real wage rigidities. Chetty and Szeidl (2007) show that consumption commitments amplify moderate-stake risk aversion. Shore and Sinai (2009) show that households who are subject to exogenously higher income risk optimally undertake larger housing commitments. Olney (1999) gives historical evidence that exposure to installment finance commitments forced households to cut back on other consumption, exacerbating welfare loss in the great Depression. The present paper highlights the tight connections between this body of work and models of reference dependence and habit formation.

The remainder of the paper is organized as follows. Section 2 shows how adjustment costs generate reference-dependent preferences. Section 3 introduces the model with heterogenous agents. In section 4, we establish the main result showing the equivalence between the aggregated commitment model and models of habit formation. Section 5 presents the applications. Section 6 concludes.

2 An Adjustment-Cost Model of Reference-Dependent Preferences

In this section, we develop a model in which consumption commitments generate an indirect utility function that exhibits reference dependence. Because our primary goal is to characterize the endogenous dynamics of reference points, we consider a highly specialized model where these dynamics can be analyzed analytically. Like habit formation and the Koszegi-Rabin framework, adjustment-cost based reference points can also be incorporated into richer models and analyzed using numerical methods.

2.1 Setup

We study an economy with a continuum of consumers to facilitate our analysis of aggregate dynamics. Each agent maximizes expected lifetime utility given by

$$\mathrm{E}\int_0^\infty e^{-\rho t}\left(\kappa\frac{{a_t}^{1-\gamma}}{1-\gamma}+\frac{{x_t}^{1-\gamma}}{1-\gamma}\right)\mathrm{d}t \tag{1}$$

where ρ is the discount rate and κ measures the relative preference for adjustables. There are two components of consumption: commitment (x) and adjustable (a). The goods x and a may be viewed as different components of the same consumption good c – e.g., adjustable margins of housing such as the quality of a kitchen vs. fixed components such as the amount of land – or as two distinct goods. We normalize units so that the relative price of the two goods is one. Adjustment of commitment consumption on a date t has a monetary cost of $\lambda_1 x_{t-} + \lambda_2 x_t$ where $\lambda_1, \lambda_2 \geq 0$ and at least one of them is positive. This cost could reflect the transaction cost inherent in changing consumption of illiquid durables such as houses, cars, or appliances or the cost of renegotiating service contracts (Attanasio 2000, Eberly 1994, Grossman and Laroque 1990, Bertola and Caballero 1990).[6] The switching cost may also arise from consumption plans, attention costs, mental accounts, or the difference between the planner and doer self (Ameriks, Caplin and Leahy, 2003, Reis, 2006, Thaler, 1999, Fudenberg and Levine, 2006). In this case, the adjustment costs can be interpreted as the psychological cost of changing plans or the contemplation costs required to make new choices (Ergin 2003).

When $\kappa \to \infty$, this framework converges to a neoclassical model without adjustment costs, and when $\kappa = 0$ we obtain a model with only commitment consumption, as in Grossman and Laroque (1990). Because utility is time-separable, γ measures the elasticity of intertemporal substitution as well as relative risk aversion for an individual who is free to adjust both x and a. We use this functional form to make the evolution of reference points tractable. In this sense, our analysis is exploratory; however, we believe that the intuition behind our main results extend to more general specifications.[7]

We ignore imperfections in financial markets, and allow the agent to invest in a bond with a constant instantaneous riskfree return r, so that the face value of the bond evolves as

$$\frac{dB_t}{B_t} = rdt. \tag{2}$$

We also allow two types of risky investments, both with i.i.d. returns. The instantaneous return of the stock market is

$$\frac{dS_t}{S_t} = (r + \pi)dt + \sigma dz_t \tag{3}$$

[6] More recently, several studies have examined state-dependent models with two consumption components, one freely adjustable and one that is costly to adjust (Flavin and Nakagawa 2003, Fratantoni 2003, Li 2003).

[7] For example, we have shown that Cobb-Douglas preferences also permit a habit representation result. In that case, the representative consumer has proportional habit utility (as in Abel, 1990) over adjustable consumption.

where z_t is a standard Brownian motion that generates a filtration $\{\mathcal{F}_t, 0 \leq t < \infty\}$, π is the expected excess return, and σ is the standard deviation of asset returns. Households also face idiosyncratic risk in the form of a household-specific risky investment opportunity. This background risk can be thought of as entrepreneurial investment or labor income risk (where "investment" is investment in human capital). The return of household i's entrepreneurial investment is given by

$$\frac{dS_t^{E,i}}{S_t^{E,i}} = (r + \pi_E)dt + \sigma_E dz_t^i$$

where the z^is are standard Brownian motions uncorrelated across households. Each household is free to invest or disinvest an arbitrary amount into his private asset at any time.

We make the standard assumption that

$$\rho > (1-\gamma)\, r + \left[\frac{\pi^2}{2\sigma^2} + \frac{\pi_I^2}{2\sigma_I^2}\right]\frac{1-\gamma}{\gamma},$$

which ensures that in the case where adjustment costs are zero, in the optimal policy expected consumption utility grows at a slower rate than the discount rate, generating finite lifetime utility for the consumer.

We denote the wealth of household i at time t by w_t^i. It is useful to think of this wealth as a combination of current wealth and the present value of expected future income. Because the agent can transfer resources over time, this distinction is irrelevant for solving the model. However, it matters for interpretation, in that w_t reflects both current resources and the expectation of future resources.

2.2 Households Exhibit Reference Dependent Preferences

The following proposition characterizes household behavior in this model. Note that the results in this proposition have been established in the literature for a class of models that nests our model as a special case (see e.g., Flavin and Nakagawa, 2008). The proposition below recasts these properties in terms of an endogenous reference point. Let $v\,(c, x) = u\,(c - x, x)$ be the period indirect utility over total consumption $c = x + a$ given commitments x.

Proposition 1 *[Adjustment costs generate reference-dependent preferences]*

(i) $\partial^2 v\,(c, x)\,/\partial c \partial x > 0$*: holding fixed total consumption c, higher commitments x increase marginal utility.*

(ii) The reference point is pre-determined for small but abandoned for large wealth shocks: there exist $s < S$ such that x is not adjusted as long as $x_t/w_t \in (s, S)$, but adjusted otherwise.

(iii) The reference point is determined by recent expectations: there exists $\omega > 0$ such that if at time t adjustment last occurred on date s, then $x_t = \omega \cdot w_s$.

(iv) The reference point is linked to recent consumption: there exists $\omega' > 0$ such that if at time t adjustment last occurred on date s, then $x_t = \omega' \cdot c_s$.

This proposition establishes that adjustment costs endogenously generate a reference point that captures many common intuitions that are exogenously posited in existing theories. Part (i) establishes that preferences are "reference dependent" in the sense that marginal utility (and hence also total utility) is evaluated relative to the current level of commitment consumption x_t. Intuitively, when x_t is high, a given drop in total consumption c_t results in a larger proportional reduction in a_t, driving up marginal utility more quickly. As a result, in periods when the consumer does not adjust x_t, it acts as a pre-determined reference point.

Parts (ii)-(iv) of the proposition characterize the determinants and evolution of the commitment-based reference point. Part (ii) shows that commitments are adjusted for large, but not for small shocks, consistent with the prior literature on (S,s) models. The model thus endogenously predicts that the importance of the reference point diminishes for large fluctuations, a feature which Koszegi and Rabin (2006) assume by postulating that utility has two parts, one reference dependent and the other neoclassical.

Next, (iii) shows that when the consumer updates, the choice of commitments is determined by expected lifetime wealth w_t. In periods where x_t is not changed, the current reference point is determined by expectations in the period where the reference point was last adjusted. Thus, the commitment-based reference point is set in a forward-looking manner, as postulated by Koszegi and Rabin. Because agents are generally more likely to have adjusted commitment consumption in the recent past than the distant past in an environment with shocks, this result suggests that the current reference point is largely determined by recent expectations.

Finally, part (iv) shows that the commitment based reference point is also connected to past consumption levels, as in backward-looking models of reference dependence. Intuitively, the permanent income hypothesis implies that expectations and permanent income were also the determinants of total consumption in the recent past. Because both the reference point and recent consumption levels are determined by permanent income in the recent past, the reference point is connected to past consumption, in a manner similar to models of habit formation (Constantides 1990, Campbell and Cochrane 1999).

The benefit of the adjustment cost microfoundation is that it yields an explicitly dynamic theory of the reference point, whose evolution can be characterized simply by specifying the adjustment cost. This characterization is the primary focus of our paper.

3 Aggregation in a Heterogenous Agent Economy

To facilitate the characterization of dynamics, we analyze an economy populated by many consumers. In a heterogenous agent economy, individual lumpy adjustments are smoothed out, allowing us to focus on the "average" dynamics of the reference point. This approach allows us to compare our model with habit formation, which is generally used to study the behavior of macroeconomic aggregates.

3.1 Household Dynamics

Proposition 1 characterized the optimal policy of a single agent. To analyze the dynamic implications of these policies, it proves useful to describe the household's choice of x in terms of an (S,s) band over x/a instead of x/w. Intuitively, a_t is a more convenient scaling variable because it adjusts immediately and fully to shocks, while the dynamics of w_t also takes into account expenditures on commitments which respond sluggishly. For this reason, a_t is usefully thought of as a measure of the permanent income of the agent. To obtain an (S,s) band for x/a, note that adjustable consumption a_t is a strictly increasing function of current wealth w_t, and hence the consumption function $a_t(w_t, x)$ can be used to map the (S,s) band over wealth into an (S,s) band over adjustable consumption for any given x. Since utility is homogenous of degree $1-\gamma$, the (S,s) band can be written in terms of x/a.

Define $y = \log(x/a)$. Then each household's optimal policy for the committed portion of consumption can be described by three numbers, $\{L, U, M\}$. For $y \in (L, U)$, the household does not adjust x from its prior level; as soon as y reaches L or U, the household resets x so that $y = M$. Since households have identical preferences and the model is scalable in wealth, the numbers $\{L, U, M\}$ do not vary across households in the economy. However, because of the idiosyncratic noise, households have different values of y in general – that is, they will be in different locations within their (S,s) bands. The non-degenerate cross-sectional distribution of y at each time t yields a smooth path for aggregate consumption because only a small fraction of households adjust x in response to a shock.

Given $\{L, U, M\}$, we can completely characterize the behavior of a household by the dynamics of a_t. The appendix shows that $\log a_t$ is a random walk with drift that satisfies

$$d \log a_t^i = \mu_a \cdot dt + \frac{\pi}{\gamma\sigma} \cdot dz_t + \frac{\pi_I}{\gamma\sigma_I} \cdot dz_t^i$$

where μ_a is the constant mean growth rate. The second and third terms measure how permanent income a_t responds to aggregate shocks dz_t and idiosyncratic shocks dz_t^i. Motivated by this expression, we define aggregate and idiosyncratic consumption risk by $\sigma_A = \pi/(\gamma\sigma)$ and $\sigma_I = \pi_E/(\gamma\sigma_E)$. These variables measure the standard deviation of adjustable consumption due to aggregate and idiosyncratic risk, respectively. Let $\sigma_T^2 = \sigma_A^2 + \sigma_I^2$ measure total consumption risk.

3.2 Preferences of the Representative Consumer

We now show that aggregate dynamics in the adjustment cost model coincide with those of a single-agent economy where aggregate commitments act as a habit-like reference point for the representative consumer. Let capital variables denote unconditional aggregates, so that X_t, A_t, and C_t denote aggregate commitment, adjustable, and total consumption at time t.

Proposition 2 *The aggregate dynamics of consumption are the optimal policy of a representative consumer with external habit formation utility*

$$E \int_0^\infty e^{-\delta t} \frac{(C_t - X_t)^{1-\gamma}}{1-\gamma} dt \tag{4}$$

as long as the discount rate $\delta = \rho - \frac{\pi_I^2}{2\sigma_I^2}\left(1 + \frac{1}{\gamma}\right) > 0$*, where* X_t *follow the dynamics of aggregate commitments.*

This result generalizes part (i) of Proposition 1 to the aggregate economy, establishing that marginal utility of the representative consumer is increasing in the aggregate commitment stock. The intuition for the existence of a representative consumer is that idiosyncratic shocks cancel out in the aggregation, as in Grossman and Shiller (1982). However, the presence of idiosyncratic risk increases both the mean and the variance of household consumption growth. To compensate for the increase in mean consumption growth in the aggregate, the representative consumer must be more patient than the individual households.

An interesting implication of Proposition 2 is that the functional form for the utility of the representative consumer obtained here is identical to the commonly used "additive habit" specification in the literature (Constantinides, 1990, Campbell and Cochrane, 1999). This is a consequence of the utility function (1). In this framework, the only observational difference in the aggregate between the commitment model and habit formation models comes from the dynamics of the reference point itself.

4 Dynamics of the Reference Point

We now characterize the evolution of the aggregate reference point X_t in the representative agent economy to evaluate whether the assumptions made in the existing literature are consistent with an adjustment-cost microfoundation. We begin by connecting aggregate dynamics to the evolution of the cross-sectional distribution of households, which we use to write aggregate commitments as a moving average of past shocks, with state-dependent weights. We then show that models where the reference point is an average of past consumption, such as habit formation, admit a similar moving average representation with state-independent weights. This leads us to our main result, which is to identify conditions under which the commitment model produces state-independent weights, establishing an approximate equivalence with habit formation models. Finally, we present an illustrative calibration to show how this result can be used to guide specification of habit weights.

4.1 The Cross-Sectional Distribution

Characterizing the dynamics of X_t requires keeping track of where households are in their (S,s) bands. Using the interpretation that a_t measures the permanent income of an agent, let $F(y,t)$ measure the cross-sectional distribution of adjustable consumption in the (S,s) band, and $f(y,t)$ the associated density. Informally, $f(y,t) = a_t(y)/A_t$ is the share of aggregate adjustable consumption at time t accounted for by households i for whom $y_t^i = y$. The density $f(y,t)$ can be interpreted as the cross-sectional distribution of permanent income in the (S,s) band, measured relative to aggregate permanent income.

Our first objective is to characterize how $f(y,t)$, which summarizes the state of the economy on date t, evolves over time. Let μ_A denote the instantaneous drift of A_t.

Proposition 3 *$f(y,t)$ satisfies the stochastic partial differential equation for $t > 0$ and $y \in (L,U)$*

$$df(y,t) = \left[\left(\mu_A + \frac{\sigma_I^2}{2}\right)\frac{\partial f(y,t)}{\partial y} + \frac{\sigma_T^2}{2}\frac{\partial^2 f(y,t)}{\partial y^2}\right]dt + \sigma_A \frac{\partial f(y,t)}{\partial y}dz \tag{5}$$

together with the following boundary conditions:

$$\frac{\partial f(M,t)}{\partial y}^+ - \frac{\partial f(M,t)}{\partial y}^- = \frac{\partial f(U,t)}{\partial y}^- - \frac{\partial f(L,t)}{\partial y}^+$$

$$f(U,t) = f(L,t) = 0 \text{ and } f(M,t)^+ = f(M,t)^-.$$

Aggregate commitments follow the dynamics

$$dX_t = A_t \frac{\sigma_T^2}{2} \cdot \left(f_y(L,t)(e^M - e^L) + f_y(U,t)(e^U - e^M)\right)dt. \tag{6}$$

This result is based on Propositions 1 and 2 in Caballero (1993). The first statement is a consequence of the Kolmogorov forward equation, which characterizes the evolution of marginal distributions in the presence of aggregate shocks. In our situation, the Kolmogorov equation cannot be applied directly, because $f(y,t)$ is not a marginal distribution of y_t, since it is weighted by A_t. To deal with this, we introduce a new probability measure Q that weights the sample path of y_t for each agent by his consumption share a_t/A_t. The benefit is that under Q, the density $f(y,t)$ *is* the marginal distribution of y, allowing us to apply the Kolmogorov equation directly. Switching to Q also affects drifts by the Girsanov theorem: intuitively, Q assigns higher weight to households with high a/A, which is equivalent to having a higher expected growth rate. Taking this change in drift into account yields (5). The boundary conditions follow as in Caballero (1993).

Equation (6) shows that the evolution of commitments is“ smooth” in the aggregate in the sense that it is a bounded variation process (has no dz term). This follows because the cross-sectional densities go to zero near the boundary of the (S,s) band. The total mass of agents who adjust in response to an aggregate shock of size dz is proportional to the area under the density at the boundaries, which is of order $(dz)^2 = dt$. Intuitively, because only a small mass of agents adjust on any given date, aggregate commitments evolve smoothly.

To understand the intuition for equation (5), first consider the case with no aggregate risk ($dz = 0$). Then the final term on the right hand side vanishes, and the resulting partial differential equation has a unique time-invariant solution f^*. This cross-sectional density f^* can be thought of as the "unperturbed" steady state of the economy, and is also coincides with the long term expected cross-sectional distribution of the model. In the presence of aggregate shocks, the actual cross-sectional distribution f is constantly perturbed relative to f^*, as represented by the dz term in (5); but in the long term the system returns to f^* in expectation.

Figure 1 illustrates these results. The top panels plot the steady-state distribution f^* in two environments: one with high aggregate risk and low idiosyncratic risk and the other with low aggregate and high idiosyncratic risk. The bottom panels show the actual cross-sectional distribution sampled twenty times from simulating the two environments. The actual distributions are similar in shape to the steady state distribution. The similarity is particularly strong when idiosyncratic risk is high relative to aggregate risk. This follows because idiosyncratic risk forces the distribution to converge towards f^*, while aggregate risk pushes it away. When σ_I/σ_A is high, the first effect is relatively stronger, and hence f spends more time in the vicinity of f^*. This observation plays a key role in our approximation result below.

4.2 Impulse Responses and a Moving-Average Representation

To connect the dynamics of X_t to exogenous reference point models, it is helpful to develop a moving average (MA) representation for X_t. This representation summarizes the dynamic response of X_t to past aggregate shocks. Because our interest is in fluctuations, we focus on the de-trended processes $\overline{A}_t = e^{-\mu_A t} A_t$, which is a martingale, and $\overline{X}_t = e^{-\mu_A t} X_t$. It is useful to think of $\overline{A}_t$ as summarizing past aggregate shocks in the economy up to time t.

Definition 1 *The impulse response function of the commitments model in state f is the function*

$$\xi\left(t|f\right) = \frac{\partial E_0\left[\overline{X}_t|f\right]}{\partial \overline{A}_0}.$$

Thus $\xi\left(t|f\right)$ measures the effect of an aggregate shock at date zero on consumption commitments t periods later, given that the initial cross-sectional distribution is given by f. The Appendix shows that $\xi\left(t|f\right)$ is well-defined. Impulse responses depend on the initial distribution f: when many households are on the verge of downsizing, a negative aggregate shock will reduce commitments at a faster rate. Figure 2 plots impulse-responses in our model in four environments (assuming $f = f^*$). As $t \to \infty$, these impulse responses converge to a limit (normalized to one in the figures), which corresponds to full adjustment to the initial aggregate shock. For finite t, the figures indicate partial adjustment. Higher risk leads to more rapid convergence, as commitments are updated more quickly.

The impulse response function allows us to make explicit the dependence of X_t on past aggregate shocks.

Proposition 4 *De-trended aggregate commitments admit the moving average representation*

$$\overline{X}_t = \int_0^t \xi\left(t-s, f\left(s\right)\right) d\overline{A}_s + E_0 \overline{X}_t. \tag{7}$$

As we show below, this MA representation is the key diagnostic in analyzing the dynamics of X_t. The result is intuitive: the current level of $\overline{X}_t$ equals it's ex ante expectation plus the sum of the effects of aggregate shocks between date 0 and date t, accounting for partial adjustment to shocks using the impulse response function. The proof follows from Ito's lemma combined with the observation that the impulse response is smooth. We interpret (7) as a "state-dependent MA representation" for commitments, where the coefficients $\xi\left(t-s, f\left(s\right)\right)$ depend on the state of the economy through f.

4.3 Habit Models and a State-Independent MA Representation

A leading special case of the moving-average representation in (7) is where the weights ξ are state-independent (i.e., do not depend on the cross-sectional distribution f). This is an environment where the impulse response to a shock does not depend on history. We now show that this special case *coincides* with reduced-form habit models in which X_t is specified as an average of past consumption with "state-indendent" weights that only depend on the time lag. This result is intuitive: if habit is a linear function of past consumption, then it should be expressible as a linear function of shocks to past consumption as well. This observation allows us to compare dynamics in the commitment and habit models by analyzing their respective MA representations. We now introduce the representative agent habit model and develop the MA representation.

Habit model. Consider a representative agent economy in which external habit preferences are given by (4), and where the habit stock is exogenously determined as

$$X_t^h = o^h(t) X_0^h + \int_0^t \zeta^h(t-s) C_s^h ds \tag{8}$$

with weights ζ^h and o^h which are exogenous locally integrable functions asymptoting to zero. Throughout, we follow the convention that the superscript h refers to the representative agent habit model. We assume that the habit consumer has access to the same stock and bond investment opportunities given in equations (3) and (2). Our habit model is therefore a variant of Constantinides (1990). Since the shock processes are identical, we can think of the habit and commitment models as being defined on the same probability space. It is a direct consequence of the Euler equation that in the optimum, the "surplus" consumption $C_t^h - X_t^h$ for the habit agent follows the same path as A_t in the commitments model.[8] Thus, A_t keeps track of aggregate shocks to marginal utility in both economies.

Moving average representation. To obtain an MA representation in this model, we first express the habit stock as a weighted average of A_t. Lemma 6 in the appendix shows the existence of locally integrable functions θ and θ_0 such that

$$X_t^h = \int_0^t \theta(t-s) A_s ds + \theta_0(t) X_0^h. \tag{9}$$

This follows essentially because C, X and A are linked by an accounting identity, and hence any linear representation of X in terms of C can also be written as a linear representation in terms of A. Detrending the habit variable by letting $\overline{X}_t^h = e^{-\mu_A t} X_t^h$, integration by parts yields

$$\overline{X}_t^h = \int_0^t \xi^h(t-s) \cdot d\overline{A}_s + E_0 \overline{X}_t^h \tag{10}$$

where $\xi^h(u) = \int_0^u e^{-\mu_A v} \theta(v)\, dv$ is absolutely continuous with respect to the Lebesgue measure. Equation (10) is an MA representation for the detrended habit stock with state-independent weights in the reduced-form habit model. We have thus established that habit models have fixed-weight MA representations.

4.4 Equivalence Result: A Fixed-Weight Representation in the Commitments Model

The results above imply that the central difference between fixed-weight habit and commitment models comes from the state-dependent nature of impulse-responses in the latter case. Hence, the commitment model of reference dependence is identical to existing habit models when the evolution of aggregate commitments is determined by a fixed-weight average of past consumption. We establish the main equivalence result by showing that aggregate commitments evolve *approximately* according to a fixed weight specification when the ratio of idiosyncratic to aggregate risk is high. To establish this result, we first introduce a fixed-weight model that generates reference point

[8] This observation forms the basis of the proof of Proposition 2.

dynamics that match the evolution of commitments *on average.*

Definition 2 *A fixed-weight habit model X_t^h matches the steady state impulse response of commitments if $\xi^h(t) = \xi(t, f^*)$ for all t.*

In words, we focus on the habit model that has the same impulse responses as the commitment model in its "unperturbed" steady state f^*.[9] This definition pins down all MA coefficients in (10), and hence the fixed-weight habit model satisfying the definition is uniquely defined. We denote the impulse-response weights by $\xi^*(u) = \xi(u, f^*)$ and the habit model by X_t^{h*}.

Main result. Our equivalence result holds when the ratio of idiosyncratic (σ_I) to aggregate (σ_A) consumption risk is large. Since both of these parameters are endogenous, we study sequences of exogenous parameters such that the implied ratio σ_I/σ_A goes to infinity. We explain why σ_I/σ_A drives the result in the discussion below. Consider a sequence of models Θ^n such that, as $n \to \infty$, the following properties hold: 1) $\sigma_I^n/\sigma_A^n \to \infty$; 2) γ, κ and $\overline{\lambda}_i$ remain fixed; 3) r^n stays bounded away from zero; 4) μ_A^n remains bounded; and 5) r^n/ρ^n is bounded away from zero and infinity. An example of such a sequence is when $\pi^n = 1/n$, while all other exogenous parameters stay constant. In this sequence, $\sigma_A^n \to 0$.

Theorem 1 *For any sequence of models Θ_n specified above and any $p \geq 1$,*

$$\limsup_t \left\| \frac{X_t - X_t^{h*}}{A_t} \right\|_p = o\left(\frac{\sigma_A}{\sigma_I}\right).$$

The left hand side of the expression measures the distance between aggregate commitments X_t and habit in the matching fixed-weight model X_t^{h*}, rescaled by a measure of the aggregate economy A_t. Since these quantities are stochastic, we use the L_p norm to measure distance, defined as $\|Y\|_p = [\mathrm{E}Y^p]^{1/p}$ for any random variable Y. The small order $o(.)$ on the right hand side shows the accuracy of the approximation: the distance between the two models becomes an arbitrarily small *share* of σ_A/σ_I when this ratio goes to zero. The interpretation is that fixed-weight habit provides a highly accurate, "better than first-order" approximation. For example, along a sequence where $\sigma_A \to 0$, the difference between commitments and the fixed-weight model goes to zero even *relative to* σ_A: when the size of aggregate shocks shrinks, the approximation error becomes small compared to the these shocks. Similarly, when the magnitude of idiosyncratic risk

[9] An alternative definition would be to choose fixed weights that generate dynamics which minimize a mean-squared error. Our formal results hold for that specification as well, and in simulations the differences between the two models are small.

grows, the distance between the two models goes to zero at a rate that is faster than the growth in σ_I.

Simulations presented in Figure 3 illustrate the theorem.[10] The figures use calibrated versions of the model to plot the evolution of X_t and X_t^{h*} in four environments, where σ_I and σ_A equal to either 5% or 10%. Parameters are set to keep μ_A fixed across these four specifications, allowing us to use the same sequence of aggregate shocks in all cases. In Figure 3a, the evolution of commitments (red, dashed) and matching habit (green, solid) are shown, along with the realized path of permanent income A_t (blue). The figures show that the difference between the two models is small in most periods, particularly when idiosyncratic risk is high (right panels) and when aggregate risk is low (bottom panels). Figure 3b further highlights this by plotting the ratio X_t^{h*}/X_t, which is closer to 1 when σ_I/σ_A is larger.

The general intuition underlying Theorem 1 is that when consumers face a high degree of idiosyncratic risk, the cross-sectional distribution is usually close to its steady state. Hence aggregate shocks generate the same pattern of adjustment in most periods, resulting in impulse response weights that are almost constant over time. The proof of the theorem involves several technical steps that are presented in the appendix; here we focus on the basic argument. The key is to analyze both models using their MA representations. Differencing (7) and (10) yields

$$\begin{aligned}\overline{X}_t - \overline{X}_t^{h*} - E_0\left[\overline{X}_t - \overline{X}_t^{h*}\right] &= \int_0^t \left[\xi^*(t-s) - \xi(t-s, f(s))\right] \cdot d\overline{A}_s \\ &= \int_0^t \left[\xi^*(t-s) - \xi(t-s, f(s))\right] \cdot \sigma_A \cdot \overline{A}_s dz_s\end{aligned}$$

where we use $d\overline{A}_s = \overline{A}_s \sigma_A dz_s$. Focusing on the final integral, consider a sequence of models Θ_n where the level of aggregate risk $\sigma_A \to 0$. Since the integrand involves σ_A, its value goes to zero for each s – as aggregate shocks become small, both models will stay close to their unconditional expectation. But the equation also reveals an important additional effect. As σ_A/σ_I becomes small, much of the shock each household experiences is idiosyncratic. This pushes the cross-sectional distribution f close to its unperturbed steady state f^*, because the force pushing for convergence toward f^*, which is proportional to σ_I, becomes stronger relative to the force of divergence, determined by σ_A.[11] As result, f and f^* are close in most periods (see Figure 1). This in turn implies that $\xi^*(t-s) - \xi(t-s, f(s))$ is typically small: when the system is close to the

[10] The parameters are given in the notes to the figures.
[11] This mechanism is labeled the "attractor effect" by Caballero (1993).

steady state, its impulse response is also close to the steady state shape. Thus $X - X^h$ is on average small even relative to σ_A.

A similar argument applies when $\sigma_I \to \infty$. The baseline effect of higher idiosyncratic risk is higher adjustment frequency. This implies that aggregate shocks are absorbed at a faster rate, so that the amount of aggregate risk that accumulates during a typical non-adjustment period is small. This mechanism reduces the distance between the two models at a rate proportional to σ_H/σ_I. At the same time, a high σ_I/σ_A also implies that the cross-sectional distribution is usually close to its steady state, so that absorption of aggregate shocks follows approximately the same pattern at all times, generating faster convergence.

The mechanism described here is also illustrated in the bottom panel of Figure 1. As noted above, there is much more "variance" in the evolution of the cross-sectional distribution in the left panel (low σ_I/σ_A), because the forces of divergence are stronger. This creates fluctuations in the impulse-response across periods, producing behavior that diverges from a fixed-weight habit model. In contrast, the cross-sectional density varies much less in right panel. As a result, the impulse-responses are approximately constant, creating aggregate consumption dynamics that are very close to those produced by a fixed-weight habit specification.

The special case where σ_I/σ_A is large is not just of theoretical interest but may also be the most empirically relevant scenario, since idiosyncratic consumption risk is generally much greater in magnitude than economy-wide shocks (e.g., Deaton, 1991, Carroll, Hall, and Zeldes 1992). Coupled with the illustrative calibrations in Figure 3, this suggests that in practice, the dynamics of the reference point the commitment model are typically well approximated by state-independent weights. Hence, existing models of habit formation are likely to provide a fairly accurate reduced-form description of aggregate behavior in normal times if reference dependent behavior arises from adjustment costs. However, reduced-form habit models may provide a much less accurate description of behavior in infrequent extreme events such as crises, as we demonstrate below.

4.5 Illustrative Calibration of Habit Weights

One benefit of deriving the habit formation model from a microeconomic model of adjustment costs is that one can calibrate the habit weights by specifying the adjustment cost and the relative preference for consumption commitments. As an illustration, Figure 4 compares the habit weights generated by the commitments model with the exponential habit specification of Ryder and Heal (1973) and Constantinides (1990). The blue dashed line shows the log habit weights generated by commitments in the low aggregate, high idiosyncratic risk environment of Figure 3, where the adjustment cost equals one year's consumption value (or 2.5% of the capitalized value of the asset). As shown above, the habit model using these weights tracks the dynamics of commitments very closely in this environment. The green solid line illustrates exponential weights by plotting the log of the weights from Table 1, column 5 in Constantinides (1990). The relative preference for commitments κ is chosen so that the sum of weights is the same across the two models; this implies that the reference point is in the range of 80% of total consumption in both environments.

The figure reveals two differences between these habit specifications. First, the commitment-based model places larger weight on the recent past (last two quarters). The intuition parallels the logic emphasized in Caplin and Spulber (1987) and Golosov and Lucas (2007) for menu cost models. In the commitment environment, there are always some households at the boundary of the inaction region; in response to a shock these agents update immediately, and when they do, they change their consumption discontinuously, resulting in quick initial adjustment in the aggregate. Second, the commitment model has lower consumption weights for intermediate horizon of up to about 3 years. This is because the mass of households in the middle of the inaction region need a longer time to update.

These calibration results show that although commitments provide a foundation for fixed-weight habit specifications, they do not necessarily generate the same weights that are used in applications. The commitment-based reference point puts relatively large weight on the recent past, offering a middle ground between models where habit equals last period's consumption (Fuhrer 2000, Christiano, Eichenbaum, Evans 2005) and models with weights on past consumption that decay exponentially (Ryder and Heal 1973, Constantinides 1990).

5 Applications

Our analysis thus far has established that adjustment costs provide foundations for many features of reference-dependent models currently used in the literature. But perhaps a more important lesson is that the endogenous evolution of reference points leads to different predictions in many settings. We demonstrate this lesson by comparing the predictions of our micro-founded model of habit with those of reduced-form habit models in some common applications.

5.1 Consumption Dynamics

Two well-documented features of aggregate consumption behavior are excess sensitivity and excess smoothness to shocks (Flavin 1981, Deaton 1987). One major reason for using habit preferences in applied macroeconomic models is that they generate such delayed consumption responses (Fuhrer 2000). In this section, we show that the commitment model of reference points also creates these patterns, but with some important caveats that challenge existing results.

Consider the following empirical regression specification for consumption growth:

$$\log(C_{t+\Delta t}) - \log(C_t) = \alpha + \beta_1 \cdot [\log A_{t+\Delta t} - \log A_t] + \varepsilon. \tag{11}$$

Following Flavin (1981), we say that consumption is excessively smooth if $\beta_1 < 1$ for some $\Delta t > 0$, i.e. if consumption fails to respond to contemporaneous shocks to the extent predicted by the permanent income model. Next, let $s_1 < s_2$ and consider the regression

$$\log(C_{t+s_2}) - \log(C_{t+s_1}) = \alpha + \beta_2 \cdot [\log A_t - \log A_{t-\Delta t}] + \varepsilon. \tag{12}$$

We say that consumption is excessively sensitive if current consumption responds to past shocks to permanent income, i.e. if there exist $0 < s_1 < s_2$ and $\Delta t > 0$ such that $\beta_2 > 0$.

Proposition 5 *In the commitments model, consumption is both excessively sensitive and excessively smooth.*

Excess smoothness follows because commitments do not respond at all to instantaneous shocks, generating $\beta_1 \approx A_t/C_t < 1$ for small Δt in the regression specified in (11). Excess sensitivity is an implication of the fact that eventually, households do adjust their commitments, and hence β_2 converges to 1 when $s_1 \to 0$ and $s_2 \to \infty$. The shape of delayed adjustment is also illustrated in Figure 2, which plots the normalized steady-state impulse response of commitments. Thus, our model suggests that both the sluggishness and sensitivity of consumption may be consequences of adjustment costs that delay updating.

Large shocks and changing correlations. While the commitment and habit models generate approximately similar dynamics as shown in Theorem 1, this similarity breaks down in periods where the economy is hit by extreme shocks. For example, the simulation in Figure 3 shows that the difference between the two models is larger on dates with large shocks to permanent income (e.g. period 71). To formalize this observation, we first define "extreme times" in the model.

Given values of $\Delta A > 0$ and $\Delta t > 0$, consider an event $B_{\Delta t}^{\Delta A}$ in which $\left|\log \overline{A}_{t+\Delta t} - \log \overline{A}_t\right| \geq \Delta A$. This event represents realizations where aggregate permanent income changes by more than ΔA during a Δt time period. Consider the case when Δt goes to zero while ΔA remains positive, i.e., the extreme event when a large change in permanent income is realized in a short time period. We compare the commitment model with its matching habit specification introduced in Definition 2. Let β_1^c denote the regression coefficient in equation (11) obtained in the commitment model, and β_1^h denote the corresponding coefficient in the reduced form habit model. Finally, let $\beta_{1,ext}^c$ and $\beta_{1,ext}^h$ denote the coefficients when the regressions are estimated only using the extreme event $B_{\Delta t}^{\Delta A}$.

Proposition 6 *For any $\Delta A > 0$, when $\Delta t \to 0$:*

(i) Commitments adjust more than fixed-weight habit in response to extreme shocks:

$$E\left[\left|\log X_{t+\Delta t} - \log X_t\right| \mid B_{\Delta t}^{\Delta A}\right] > E\left[\left|\log X_{t+\Delta t}^h - \log X_t^h\right| \mid B_{\Delta t}^{\Delta A}\right]$$

(ii) Extreme shocks generate less excess smoothness in the commitment model but not the habit model: $\beta_{1,ext}^c - \beta_1^c > 0$ but $\beta_{1,ext}^h - \beta_1^h \to 0$:

Part (i) of this proposition compares the expected change in the reference point in the commitment and habit models conditional on a large shock. For Δt small, the response of fixed-weight habit is approximately zero, because the impulse-response is state-independent and hence unaffected by the size of the shock. In contrast, commitments do respond, because a positive mass of households adjust in response to the extreme shock. Part (ii) shows that the correlation between consumption and permanent income increases in extreme events in the commitments model, but not in the habit model. Intuitively, as people adjust commitments in extreme events, comovement increases.

Changing correlations have implications for both theoretical and empirical studies of stabilization policy. Building on Fuhrer (2000) and Christiano, Eichenbaum and Evans (2005), habit specifications are a now a standard element of equilibrium models used to study monetary policy. If adjustment costs are the underlying source of the sluggish dynamics that are typically modeled using reduced-form habits, the predictions of these models will be invalid in extreme periods. Since periods of crisis are often of greatest interest from a policy perspective, models based on reduced-form habits may not be adequate to evaluate monetary policy. As a counterpart to these models, empirical work often relies on linear vector autoregressions (VARs) estimated using historical time series to evaluate policies. The proposition shows that the coefficients of these linear models can break down in crises, when people make costly adjustments to abandon their reference points.

5.2 Welfare Costs of Shocks

Thus far, we have focused on positive differences between the adjustment-cost based and other models of reference points. We now turn to explore differences in normative implications. One benefit of the adjustment-cost based reference point model is that it offers a natural welfare measure. In contrast, in habit and prospect theory models, the appropriate measure of welfare is open to debate – for instance, should habit consumption be included in welfare calculations?

An Example. To explore the welfare cost of shocks in a simple way, we consider consumers' willingness to pay to avoid a one-time, unanticipated wealth shock. To build intuition, we first focus on the case with no aggregate or idiosyncratic risk, where the equivalent habit model can be described directly. Consider a commitment economy in which the excess returns $\pi = \pi_E = 0$ and the interest rate $r = \rho$. In this setting, households face no risk and have zero consumption growth ($\mu_a = \sigma_T = 0$). As a result, agents either adjust commitments immediately at $t = 0$ or never do so. The habit model that matches this consumption pattern (as in Definition 2) is simply one in which the habit stock remains unchanged at the level of commitment x forever.[12]

To compare the welfare cost of unanticipated wealth shocks in these two models, consider Figure 5 which plots the value functions of the commitment and habit agents when $\pi = \pi_E = 0$ and $r = \rho$. As long as it remains optimal for the commitment agent not to move, the two value functions are completely parallel.[13] In this range, all changes in wealth are absorbed by adjustable consumption, and hence the welfare implications of the two models are identical. However, for large shocks, the commitment agent adjusts on both consumption margins, while adjustment of the habit stock is not permitted. As a result, large shocks have a higher welfare cost with habits than with commitments. Intuitively, commitment-based habits absorb large shocks, dampening their welfare cost.

Formal Result. To establish this intuition in a more general setting, we consider an unanticipated wealth shock at time t that hits with probability q and reduces total wealth by a share b, and compare the premium agents in the two models are willing to pay to avoid this risk. Consider the commitment economy in it's unperturbed steady state where all agents face this shock, and

[12] This is also the limit that obtains when we take the matching habit model defined in the previous section and reduce risk and growth to zero.

[13] There is a difference in the *level* of utility because the habit agent does not derive utility from commitment consumption. The figure abstracts away from this effect by shifting the value function of the habit agent vertically.

contrast it with the matching habit model where the shock affects the representative agent. Define the risk premium to be dollar amount that agents are collectively willing to give up in excess of the expected value to avoid this risk. The proportional risk premium $\Pi(q,b)$ is the risk premium normalized by total wealth in the economy.

Proposition 7 *If $\lambda_1 = 0$ but $\lambda_2 > 0$, then as $b \to 1$,*

(i) the proportional risk premium in the fixed-weight habit economy exceeds that in the corresponding commitment economy: $\Pi_h(q,b) > \Pi(q,b)$

(ii) the portfolio share of stocks in the commitment economy following the negative shock exceeds that in the habit economy.

Part (i) of this result implies that habit agents are more averse to large shocks than are commitment agents. Following a big shock, the reference point in the commitment model adjusts immediately, mitigating the impact of the shock. In contrast, reduced-form habits adjust sluggishly for shocks of all sizes, and thus agents suffer greatly when hit by a large shock in such models.[14] These differences in the welfare cost of shocks imply differences in the risk appetites of the commitment and the habit economies following a negative shock. Intuitively, habit agents suffer greatly from a large shock because marginal utility rises rapidly as the size of the shock increases. This also implies that following a large negative shock their marginal utility is highly curved, and hence they are very averse to any additional risk, leading to reduced stockholdings. In the commitment economy, most agents adjust after a big negative shock, reducing the impact of the shock on marginal utility and stock shares.

Figure 5 illustrates these results by plotting the ratio of the proportional risk premium in the habit and the commitment model as a function of shock size b, with $q = 1\%$. The blue line is generated by an environment with low idiosyncratic risk. As shown above, in this environment consumption responds sluggishly to shocks on average, and hence the matching habit model that has a highly persistent habit stock. As a result, the habit agent is far more averse to big shocks than the corresponding commitment agent. The green line shows the ratio of risk premia in an economy with high idiosyncratic risk. Here habit is less persistent, and hence the risk premia are similar for a wide range of shocks in the two models.

[14] The assumption that $\lambda_1 = 0$ guarantees that when moving, the commitment agents can get rid of all precommitments. Otherwise, even when moving they would still have promised expenditures of $\lambda_1 X_{t-}$, which behave like sluggish habits. In simulations, we find that unless λ_1 is very high, the conclusion of the proposition is unaffected. Intuitively, moving costs are much smaller than habit expenditures.

One lesson from this application is that a reduced-form habit model that matches observed dynamics of consumption well *on average* may nevertheless yield misleading conclusions about the welfare costs of certain shocks, particularly large shocks. Even if consumption is highly persistent for typical shocks, agents may not be extremely averse to big fluctuations because they can adjust their reference points by paying a fixed cost. This simple observation about changes in reference points could potentially have significant policy implications. For instance, the optimal size of social insurance programs that insure large, long-term shocks such as disability or job displacement may be smaller than predicted by analyses using habit models such as Ljungqvist and Uhlig (2000).

5.3 Comparative Dynamics and Policy Analysis

The preceding applications have shown that reduced-form habit models and a model of habit built from microfoundations of adjustment costs make different predictions about extreme shocks. We now show that even absent extreme events, the two models make different predictions about the effects of changes in policy or the economic environment. This is because the weights that determine the speed of adjustment are exogenous and unaffected by policy or environmental changes in reduced-form habit models. In contrast, with commitment-based habit, the adjustment of reference points is determined by households' optimizing behavior, and therefore changes with the parameters of the environment.

To illustrate the implications of this observation, we first define a measure of the responsiveness of the reference point to shocks. Let $T(p, f) = \inf_t \{\xi(t|f) \geq p \cdot \overline{x}\}$ denote the time required for the reference point to adjust in expectation a share p to a unit shock to permanent income.[15] By definition, in a fixed-weight habit model $T(p)$ is pinned down by the exogenous weights and hence remains constant when other parameters are varied. Table 1 reports $T(p|f^*)$ for the commitments model when $p = 0.25$, 0.5 and 0.75 for various parameters. In the top panel, the adjustment cost equals one year's consumption value of the commitment good, or 1% of its capitalized value with a riskfree rate of 1%. The first row shows that when $\sigma_A = \sigma_I = 10\%$ and $r_f = 1\%$, it takes on average 1.7 years for 50% of full adjustment of the reference point to occur. The next three rows illustrate the effect of reducing σ_A or σ_I, changing r_f so that expected consumption growth remains unchanged in these comparisons. The table shows that reducing either idiosyncratic or aggregate risk results in a slower response to shocks. The intuition is that higher risk forces consumers to update their commitments more frequently, allowing aggregate shocks to get absorbed

[15] Here, $\overline{x}$ denotes the steady state ratio of commitments to adjustables, so that $\xi(t)/\overline{x}$ asymptotes to one.

into the reference point more quickly. Comparing the first and last rows in the top panel shows the effect of higher consumption growth generated by a higher safer return. Faster growth again leads to faster updating of the reference point, as agents adjust consumption more frequently in a growing economy. The bottom panel of the table shows that for a higher adjustment cost (5% of the capitalized value of the commitment good), the reference point responds in a more sluggish fashion, but the effects of risk and growth remain similar. Such changes in the evolution of the habit stock in response to economic conditions would not be captured by a reduced-form habit model, in which the speed of adjustment is fixed.

Sluggish adjustment and the welfare cost of shocks. The preceding comparative dynamics imply that aggregate shocks have shorter-term effects in a commitment economy with high idiosyncratic risk, because agents have more opportunities to adjust for other reasons. This observation also suggests that aggregate shocks can have lower welfare costs in such environments. To formalize this idea, we again focus on the risk premium $\Pi(q,b)$. Consider a sequence of economies Θ^n with $n = 1, 2, ...$ where the excess returns $\pi^n = \pi_E^n = 1/n$ and $r = \rho$. Along this sequence, σ_I, σ_A, μ_a and μ_A all go to zero at a rate of $1/n$. When n grows large, this economy converges to the special case analyzed earlier, where households face no risk and have zero consumption growth, which we denote by Θ^*. By comparing agents in Θ^n versus Θ^*, we can analyze how the level of risk and growth affects behavior and welfare in the commitments model. To ask the same question in the habit model, we consider the habit specification matching Θ^*, where the habit stock is permanently fixed at x_0, and explore the effect of adding aggregate risk and growth by setting $\pi^n = 1/n$.

Proposition 8 *(i) For any $p > 0$, $T^n(p|x_0)$ is finite and $\lim_{n\to\infty} T^n(p|x_0) = \infty$ in the commitment model. In the habit model, $T^{h,n}(p|x_0) = \infty$ for all n.*

(ii) Assume that $\lambda_1 = 0$ but $\lambda_2 > 0$. For any consumer and $b > 0$ sufficiently small, in the commitment model the risk premium $\Pi^n(q,b) < \Pi^*(q,b)$, while in the habit model $\Pi^{h,n}(q,b) = \Pi^{h*}(q,b)$.

Part (i) of this proposition shows that in the commitment model, with positive risk and growth (n finite) adjustment does take place. As $n \to \infty$, adjustment occurs at vanishingly small rates, so that the expected time to adjustment converges to infinity. In contrast, in the habit model, the presence of risk and growth does not affect adjustment of the habit stock, which remains constant permanently. Part (ii) explores the welfare implications of these differences. With commitments, risk and growth reduce the risk premium $\Pi(q,b)$: since agents adjust for other reasons, a shock can be partly absorbed by commitments. Because this possibility is absent in the reduced-form habit model, there the risk premium is unaffected by changes in risk or growth.[16] The prediction that background risk can *reduce* risk-aversion when agents have reference-dependent preferences is also obtained by Koszegi and Rabin (2007). The mechanism for their result is that background risk has a smoothing effect on the kink of gain-loss utility. In contrast, the mechanism in our model is that background risk allows for more frequent updating, reducing the welfare cost of additional risk.

One practical implication of Proposition 8 is that recessions may be longer and more costly in economies with substantial social insurance against idiosyncratic risk, because people have weaker incentives to change their plans and commitments. In contrast, an economy with more idiosyncratic risk responds faster to aggregate shocks, because agents update frequently for other reasons. Hence, expanding social insurance programs in order to reduce households' exposure to idiosyncratic risk may increase the welfare cost of aggregate shocks by delaying adjustments to a given shock. This result in not obtained in existing models of habit because the degree of risk has no bearing on the weights that determine the evolution of the habit stock.

The more general lesson of this application is that the dynamics of the commitment-based reference point respond to policy changes in systematic ways that are not captured in a reduced-form habit specification. This point is analogous to the Lucas (1976) critique, and calls for caution in using reduced form models of reference dependent preferences in policy analysis because the endogenous response of the evolution of the reference point are not taken into account.

6 Conclusion

The results in this paper provide both a critique and synthesis of the literature on reference dependent preferences and habit formation. We have built a theory of reference dependent preferences based on adjustment costs in consumption that synthesize the intuitions of existing models of reference dependence. In particular, our theory predicts that reference points are determined by both recent expectations and recent consumption patterns, and become less relevant when agents face large shocks. In the special case where idiosyncratic risk is prevalent, the model provides foundations for precisely the fixed-weight habit specifications that are most widely used in the existing literature and can be used to guide the specification of weights in such models.

Although our results indicate that the preference specifications used in the existing literature

[16] As before, $\lambda_1 = 0$ guarantees that when adjusting, the agent can get rid of all commitments.

can be viewed as convenient reduced forms, they also reveal that the predictions of these reduced-form models may be inaccurate in some cases. In applications such as the analysis of aggregate consumption dynamics and the welfare cost of shocks, the reduced-form habit model and the model we develop here deliver starkly different predictions in some domains of the parameter space. This is because the evolution of the reference point is endogenous to policies and the types of shocks that agents face. Hence, in some applications, it may be necessary to build a model of reference dependence from the foundations of adjustment costs rather than directly employing a convenient reduced-form. More generally, these findings underscore the importance of modeling the foundations of non-standard preferences, and serve as a call for further theoretical and empirical research on the sources of reference-dependent behavior.

Appendix A: Proofs

Proofs for Section 2

Proof of Proposition 1 (i) Immediate. (ii) This follows from Flavin and Nakagawa (2008). (iii) and (iv) These claims follow from homothetic preferences.

Proof of Proposition 2. Since the only risky assets for household i are S and S^i, there exists a unique state price density associated with the household-specific private market. The following dynamics for adjustable consumption generates a state price density that prices both risky assets as well as the safe asset

$$a_t^i = a_0^i \exp\left\{\frac{1}{\gamma}\left(\frac{\pi^2}{2\sigma^2} + \frac{\pi_I^2}{2\sigma_I^2} + r - \rho\right)t + \frac{\pi}{\gamma\sigma}z_t + \frac{\pi_I}{\gamma\sigma_I}z_t^i\right\}$$

and hence must describe the optimal choice of household i. Because $a_0^i = A_0$ for all i, aggregating across i yields, by the strong law of large numbers for a continuum of agents (Sun, 1998)

$$\begin{aligned} A_t &= A_0 \exp\left\{\frac{1}{\gamma}\left(\frac{\pi^2}{2\sigma^2} + \frac{\pi_I^2}{2\sigma_I^2} + r - \rho\right)t + \frac{\pi}{\gamma\sigma}z_t\right\} \int_i \exp\left\{\frac{\pi_I}{\gamma\sigma_I}z_t^i\right\} di \\ &= A_0 \exp\left\{\frac{1}{\gamma}\left(\frac{\pi^2}{2\sigma^2} + \frac{\pi_I^2}{2\sigma_I^2}\left(1 + \frac{1}{\gamma}\right) + r - \rho\right)t + \frac{\pi}{\gamma\sigma}z_t\right\}. \end{aligned}$$

Define a new discount rate $\delta = \rho - \left(1 + \frac{1}{\gamma}\right)\pi_I^2/(2\sigma_I^2)$. Then the dynamics of aggregate adjustable consumption is given by

$$A_t = A_0 \exp\left\{\frac{1}{\gamma}\left(\frac{\pi^2}{2\sigma^2} + r - \delta\right)t + \frac{\pi}{\gamma\sigma}z_t\right\}.$$

This is exactly the dynamics of adjustable consumption that would obtain for a representative consumer with power utility over A_t and discount rate δ who can invest in the publicly traded risky and safe assets. The existence of a representative consumer obtains even though markets are incomplete because idiosyncratic shocks cancel out in the aggregation, as in Grossman and Shiller (1982). However, the presence of idiosyncratic risk increases both the mean and the variance of household consumption growth. To compensate for the increase in mean consumption growth in the aggregate, the representative consumer must be more patient than the individual households.

Proofs for Section 3

Proof of Proposition 3. Think about the dynamics of y_t^i as being driven by two Brownian motions, z_t and z_t^i. Our goal is to study the evolution of the distribution of y^i conditional on the path of z. Let Q be the measure on the space of sample paths of y that weighs paths of y_t by their share in aggregate consumption. The advantage of the measure Q is that the probability distribution of interest $F(y,t)$ can be written simply as $F(y,t) = \Pr_Q \left[y_t^i < y | A_{[0,t]} \right]$, that is, the marginal under Q of y_t^i, conditional on the path of aggregate shocks. The probability density associated with Q is

$$\frac{dQ}{dP}|_t = \frac{a_t^i}{A_t} = \exp\left[\frac{\pi_I}{\gamma\sigma_I} z_t^i - \frac{\pi_I^2}{2\gamma^2\sigma_I^2} t\right]$$

which is an exponential martingale. By the Cameron-Martin-Girsanov theorem, under Q, the process $d\overline{z}_t^i = dz_t^i - \pi_I / (\gamma\sigma_I)\, t$ is a Brownian motion. We are interested in characterizing the evolution of conditional distribution of y_t^i given a realization of the path of A under Q. Proposition 1 in Caballero derives a stochastic partial differential equation for such conditional densities. To apply this result here, recall that

$$d\log a_t^i = \frac{1}{\gamma}\left(\frac{\pi^2}{2\sigma^2} + \frac{\pi_I^2}{2\sigma_I^2} + r - \rho\right) dt + \frac{\pi}{\gamma\sigma} dz + \frac{\pi_I}{\gamma\sigma_I} dz^i = \theta dt + \frac{\pi}{\gamma\sigma} dz + \frac{\pi_I}{\gamma\sigma_I} d\overline{z}^i$$

where

$$\theta = \frac{1}{\gamma}\left(\frac{\pi^2}{2\sigma^2} + \frac{\pi_I^2}{2\sigma_I^2} + r - \rho\right) + \frac{\pi_I^2}{\gamma^2\sigma_I^2}$$

is the drift under Q. Caballero's stochastic differential equation is

$$df(y,t) = \left[\theta \frac{\partial f(y,t)}{\partial y} + \frac{\sigma_T^2}{2} \frac{\partial^2 f(y,t)}{\partial y^2}\right] dt + \sigma_A \frac{\partial f(y,t)}{\partial y} dz$$

and substituting in the above expression for θ gives the desired result. The boundary conditions follow directly from Caballero's proposition.

To derive the dynamics of aggregate commitments, note that $X_t = \int_L^U e^y f(y,t) dy \cdot A_t$ and we can use Ito's lemma to write

$$dX_t = A_t \int_L^U e^y \cdot df(y,t) \cdot dy + dA_t \cdot \int_L^U e^y f(y,t) dy + \left\langle \int_L^U e^y \cdot df(y,t) \cdot dy, dA_t \right\rangle.$$

We now evaluate each term on the right hand side. The first term is

$$A_t \int_L^U e^y \cdot \frac{\partial f(y,t)}{\partial y} \left\{ \left(\mu + \frac{\pi_I^2}{2\gamma^2\sigma_I^2}\right) dt + \frac{\pi}{\gamma\sigma} dz \right\} dy + F_t \int_L^U e^y \cdot \frac{\partial^2 f(y,t)}{\partial y^2} \frac{\sigma_T^2}{2} dt \cdot dy.$$

Integrating by parts, and using the boundary conditions shows that this term equals

$$-X_t\left(\left(\mu+\frac{\pi_I^2}{2\gamma^2\sigma_I^2}\right)dt+\frac{\pi}{\gamma\sigma}dz\right)+A_t\frac{\sigma_T^2}{2}\cdot\left(f_y(L,t)(e^M-e^L)+f_y(U,t)(e^U-e^M)\right)dt+\frac{\sigma_T^2}{2}X_tdt.$$

The second term is

$$X_t\cdot\frac{dA_t}{A_t}=X_t\left(\left(\mu+\frac{\pi^2}{2\gamma^2\sigma^2}\right)dt+\frac{\pi}{\gamma\sigma}dz\right)$$

while the third term is simply $-\pi^2/\left(\gamma\sigma\right)^2X_tdt$. Collecting terms gives the result of the proposition.

Proofs for Section 4

We present a serious of Lemmas and arguments that build up to the proof of Theorem 1 as well as the other results in Section 4. For Theorem 1, we will focus on a sequence Θ_n along which $\sigma_A\to 0$ for much of the proofs; in the end, we will show how to convert this result to a limit where $\sigma_I\to\infty$ with a clock change. Along the sequence Θ_n, some endogenous parameters of the model, such as the inaction region $[U,L]$ will also change. While we do not always explicitly indicate this in notation, we always understand those changes to be taking place.

We start by introducing a second measure change that will facilitate some of the arguments below. Recall that $\overline{A}_t=e^{-\mu_At}A_t$ is an exponential martingale. We define a probability measure R by letting, for any random variable Z_t measurable with respect to $\mathcal{F}_t$, $E^R\left[Z_t\right]=E\left[Z_t\overline{A}_t\right]$. The Girsanov theorem tells us that under R, the process $d\overline{z}_t=dz_t-\sigma_At$ is a martingale. The key advantage of this measure is that $E_0\overline{X}_t=E_0^R\left[\overline{X}_t/\overline{A}_t\right]$. This makes it easier to compute the mean and the impulse response of $\overline{X}_t$, because $\overline{X}_t/\overline{A}_t$ is a bounded process (under R as well as under P). We can also write

$$E_0\overline{X}_t=E_0^R\left[\overline{X}_t/\overline{A}_t\right]=E_0^{QR}\left[x_t/a_t\right]$$

where the superscript QR means that we also apply the measure transformation Q introduced earlier. The idea is that by applying R, we move to using the mean dynamics of $\overline{X}/\overline{A}$; and then, by also applying Q, we can focus on the mean dynamics of a single agent, albeit under a driving process with different drift.

We begin with a technical lemma that establishes the smoothness of conditional expectations of the process y_t. We start by thinking the dynamics of a new process w_t, which is a Brownian motion with some drift μ_w and variance σ_w reborn at some interior point M_w when hitting the boundaries of the interval $[L_w,U_w]$. When the parameters of this process are chosen to match those generated by our model, then w_t will have the same distribution as y_t under QR. We let

$h(y,t,\sigma_w,\mu_w,L_w,M_w,U_w) = E[e^{w_t}|w_0 = y]$. Often we just write $h(y,t)$, in which case we generally assume that the other arguments are at their values as given by the optimal policy of the model, so that $h(y,t) = E^{QR}E[e^{y_t}|y_0 = y]$.

Let $L_1 < L_2 < M_1 < M_2 < U_1 < U_2$.

Lemma 1 *$h(y,t,\sigma_w,\mu_w,L_w,U_w,M_w)$ is infinitely many times differentiable in $[L_w,U_w] \times (0,\infty) \times (0,\infty) \times [L_1,L_2] \times [M_1,M_2] \times [U_1,U_2]$.*

Proof. We start with the case where w_t is driven by a standard Brownian motion. Let $\zeta_y = \inf\{t \geq 0 : w_t \notin [L,U], w_0 = y\}$. Set $F_w(t) = \Pr[\zeta_y \leq t]$ and $\overline{h}(y,t) = E[e^{w_t} \cdot 1\{\zeta_y > t\}]$ be $h(y,t)$ killed at the boundary. Let $F_y^{(1)}(t) = F_y(t)$ and $F_y^{(n+1)}(t) = \int_0^t F_{y^*}^{(n)}(t-\tau)\,dF_y(\tau) = \int_0^t F_M(t-\tau)\,dF_y^{(n)}(\tau)$ be the the distribution of the $n+1$st exit time. Then

$$h(y,t) = \overline{h}(y,t) + \sum_{n=1}^{\infty} \int_0^t \overline{h}(M,t-\tau)\,dF_y^{(n)}(\tau) = \overline{h}(y,t) + \int_0^t \overline{h}(M,t-\tau)\,dF_y^*(\tau) \tag{13}$$

where

$$F_y^*(t) = \sum_{n=1}^{\infty} F_y^{(n)}(t) = F_y(t) + \int_0^t F_M^*(t-\tau)\,dF_y(\tau) = F_y(t) + \int_0^t F_M(t-\tau)\,dF_y^*(\tau) \tag{14}$$

is the expected number of boundary hits until t.

The transition density of the killed diffusion $p(y,y',t) = \Pr[\zeta_y > t, y_t = y']$ can be expressed as an infinite sum of normal densities (Revuz and Yor, 1992, p 106), and in particular, is infinitely many times differentiable in $[L,U] \times [L,U] \times (0,\infty)$. This implies that $\overline{h}(y,t) = \int e^{y'} p(y,y',t)\,dy'$ is infinitely many times differentiable in $[L,U] \times (0,\infty)$. The density of the first hitting time ζ_y can also be expressed in closed form as an infinite sum (Darling and Sieger, 1953), and is infinitely many times differentiable in y and t over $[L,U] \times (0,\infty)$. This, combined with (14) implies that $F_y^*(t)$ is C^∞ in $[L,U] \times (0,\infty)$. Combining these observations with (13) shows that $h(y,t)$ is also C^∞ in the $[L,U] \times (0,\infty)$ domain.[17]

We next show that h is also smooth when driven by any Brownian motion with drift and variance, and that it is smooth in the other parameters. Changing the clock of y_t scales both the mean and the variance, and is obviously a smooth transformation of $h(y,t)$ as it just scales the time argument. Shifting and rescaling the vertical axis are smooth operations that shift and

[17] Grigorescu and Kang (2002) compute the transition density of y explicitly.

rescale the triple $[L, M, U]$. Thus we only need to show smoothness in the drift and in M. The drift can be dealt with using the Girsanov theorem, which implies that the density of the killed diffusion under drift can be obtained as $p^{\mu w}(y, y', t) = p(y, y', t) \cdot \exp\left[\mu_w(y' - y) - \mu_w^2 t/2\right]$, which is clearly C^∞ in μ_w, and hence so is $\overline{h}(y, t)$. Next, the distribution of the first hitting time is $1 - F_y^{\mu_y}(t) = \int p^{\mu_y}(y, y', t)\, dy'$ is also smooth. The smoothness of h in μ_y now follows from (13). Smoothness in M follows easily from (13).

A key implication of this lemma is that h and it's various derivatives in y and t are all continuous and therefore locally bounded in $(\mu_w, \sigma_w, L_w, M_w, U_w)$. This is useful because when we take σ_A to zero along a sequence, optimal behavior changes, and hence the endogenous parameters $(\mu_y, \sigma_y, L, M, U)$ vary. By the theorem of the maximum, these parameters will all lie in some bounded open set, and even though $\sigma_A \to 0$, we have σ_y bounded away from zero, since there is positive idiosyncratic risk. The lemma thus implies that along this sequence, $h(y, t)$ and its derivatives in y and t exist and are all bounded. We will exploit this fact later in the proof.

Our next Lemma gives an MA representation of X_t and expresses the weights using the function h.

Lemma 2 *There exist functions $\xi(u, f)$ and $\xi(u, y)$ so that*

$$\overline{X}_t = \int_0^t \xi(t - s, f(s))\, \sigma \overline{A}_s dz_s + E_0\left[\overline{X}_t\right] \tag{15}$$

where

$$\xi(u, f(s)) = \int_L^U \xi(u, y) f(y, s)\, dy \quad \text{and} \quad \xi(u, y) = h(u, y) - h_y(u, y).$$

Proof. We have

$$E_s\left[\overline{X}_t\right] = \overline{A}_s \cdot E_s^R\left[\overline{X}_t / \overline{A}_t\right] = \overline{A}_s \cdot E_s^{QR}[x_t/a_t] = \overline{A}_s \cdot \int_L^U h(t - s, y) f(y, s)\, dy$$

which is a martingale in s. Computing the Ito-differential

$$d_s E_s\left[\overline{X}_t\right] = d\overline{A}_s \cdot E_s^{QR}[x_t/a_t] + \overline{A}_s \cdot \int_L^U h(t - s, y) f_y(y, s)\, \sigma_A dz_s \cdot dy$$

where we used (5) for the evolution of $f(y, s)$ and collected only the dz terms, since the ds terms must cancel by the martingale property. Equivalently,

$$d_s E_s\left[\overline{X}_t\right] = d\overline{A}_s \cdot \left(E_s^{QR}\left[x_t/a_t\right] + \int_L^U h\left(t-s,y\right) f_y\left(y,s\right) dy\right) = d\overline{A}_s \cdot \int_L^U \left(h\left(u,y\right) - h_y\left(u,y\right)\right) f\left(y,s\right) dy$$

where we integrated by parts. This equation shows the existence of ξ as well as the desired representation.

Proof of Proposition 4. We just need to show that $\xi\left(u,f\right)$ as defined in Lemma 2 equals the impulse response of Definition 1. Let $\overline{A}_0^*$ be the point at which we want to differentiate $E_0\left[X_t|f\right]$. The key is to note that we can write the conditional expectation as a function of $\overline{A}_0$ as

$$E_0\left[\overline{X}_t\right] = \overline{A}_0 \cdot E_0^R\left[\overline{X}_t/\overline{A}_t\right] = \overline{A}_0 \cdot \int_L^U h\left(t, y - \left(\log \overline{A}_0 - \log \overline{A}_0^*\right)\right) f\left(y,s\right) dy.$$

To see the logic, note that when $\overline{A}_0 = \overline{A}_0^*$, the mass of people at any point y is given by $f\left(y\right)$, and the conditional expectation given y is summarized by h. When $\overline{A}_0$ changes, the mass of these people is unaffected, and hence $f\left(y\right)$ is unchanged; but their y shifts, resulting in a change in the conditional expectation, and hence we must evaluate h at a different point.

Differentiating this in $\overline{A}_0$ gives

$$\frac{\partial E_0\left[\overline{X}_t\right]}{\partial \overline{A}_0} = \int_L^U h\left(t,y\right) f\left(y,s\right) dy - \int_L^U h_y\left(t,y\right) f\left(y,s\right) dy = \int_L^U \left[h\left(t,y\right) - h_y\left(t,y\right)\right] f\left(y,s\right) dy$$

which is exactly the definition of ξ given above. This confirms both that the impulse response is well defined, and the MA representation claimed in the proposition.

We next show that the impulse response function converges exponentially fast to its limit value. Intuitively, the impact of a permanent shock that happened in the distant past should be almost fully built into current commitments.

Lemma 3 *There exists $\overline{x}$ such that $\lim_{t\to\infty} E_0\left[\overline{X}_t\right] = \lim_{t\to\infty} \xi\left(t,y\right) = \overline{x}$. There exist $K_1, K_2 > 0$ independent of y and σ_A so that $\left|\xi\left(t,y\right) - \overline{x}\right| < K_1 e^{-K_2 t}$ and $\left|E_0\left[\overline{X}_t\right] - \overline{x}\right| < K_1 e^{-K_2 t}$ for all $\left(y, \sigma_A\right) \in \left[L,U\right] \times \left[0, \overline{\sigma}_A\right]$.*

Proof. Ben-Ari and Pinsky (2009) show that $y_t = \log\left[x_t/a_t\right]$ converges exponentially fast to a unique invariant distribution. Ben-Ari and Pinsky (2007) also show that the rate of convergence is uniformly bounded if the drift is from a bounded interval. This implies uniform convergence for all $\sigma_A \in [0, \overline{\sigma}_A]$ through a clock-change argument. Since

$$E_0\left[\overline{X}_t\right] = E_0^R\left[\overline{X}_t/\overline{A}_t\right] = E_0^{QR}\left[x_t/a_t\right],$$

it follows that $E_0\left[\overline{X}_t\right]$ converges exponentially fast to the mean $\overline{x}$ of x/a under the invariant distribution, and that this is uniform in σ_A. Recalling that $h(u,y) = E^{QR}\left[x_u/a_u | x_0/a_0 = e^y\right]$, we also have $h(u,y)$ converge at the same rate to $\overline{x}$ as $u \to \infty$, uniformly in y and σ_A. Letting $F_t^{QR}[y|y_0]$ denote the cross-sectional distribution of y_t given initial value y_0, fixing some $s < u$, we can write

$$\begin{aligned} h_{y_0}(u, y_0) &= \frac{\partial}{\partial y_0} \int_L^U h(u-s, y)\ dF_t^{QR}[y|y_0] = \int_L^U h(u-s, y)\ \frac{\partial^2 F_t^{QR}[y|y_0]}{\partial y_0 \partial y} dy \\ &= \int_L^U (h(u-s, y) - \overline{x})\ \frac{\partial^2 F_t^{QR}[y|y_0]}{\partial y_0 \partial y} dy \end{aligned}$$

where at the last step we used that $\partial^2 F_t^{QR}[y|y_0]/\partial y_0 \partial y$ integrates to zero in y. By the arguments of Lemma 1, $\partial^2 F_t^{QR}[y|y_0]/\partial y_0 \partial y$ is bounded, while $h(u-s,y) - \overline{x}$ converges exponentially fast to zero; hence so does the integral.

The next result will be used in the proof of Theorem 1 to establish that when σ_A is small, the impulse responses of the two models are typically close. We let f^* denote the invariant distribution of y under Q, which is also the long run average cross-sectional distribution of the commitments model.

Lemma 4 $\limsup_{t\to\infty} \left\|\sup_y |F(y,t) - F^*(y)|\right\|_p$ *converges to zero as* $\sigma_A \to 0$.

Proof. We know that EF converges to F^* uniformly in y. Fix $\varepsilon > 0$ and pick s so that for all $t > s$, $|EF_t - F^*| < \varepsilon/8$ for all initial conditions and for all σ small enough. Consider the rectangular set $[-\kappa, \kappa] \times [t-s, t]$, and let G_κ denote the event when the realization of $\log \overline{A}_u - \log \overline{A}_{t-s}$ for $u \in [t-s, t]$ is in this set. Let $F\left(y, t, \overline{A}_{[t-s,t]}, y_s\right)$ denote the distribution of y_t under Q when started at y_s in s, and when the realization of aggregate shocks is given by $\overline{A}_{[t-s,t]}$. We then have that $\left\{\sup_{y_t, y_s} \left|F\left(y, t, \overline{A}_{[t-s,t]}, y_s\right) - F\left(y, t, \overline{A}'_{[t-s,t]}, y_s\right)\right| \,\middle|\, \overline{A}_{[t-s,t]}, \overline{A}'_{[t-s,t]} \in G_\kappa\right\}$ goes to zero as $\kappa \to 0$: two sufficiently close paths of aggregate consumption generate cross-sectional distributions that are themselves close. This is because the share of people for whom the two aggregate paths result in sufficiently different behavior goes to zero. Take κ small enough so that this quantity is less than $\varepsilon/8$. For any fixed κ we can pick σ small enough so that $\Pr\left[\overline{A}_{[t-s,t]} \in G_\kappa\right] > 1 - \varepsilon/8$. This implies that $|E_s F_t - E[F_t | f(s), G_\kappa]| < \varepsilon/4$. Combining these bounds, for $\overline{A}_{[t-s,t]} \in G_\kappa$ we have

$$\left|F\left(y,t,\overline{A}_{[t-s,t]},f(s)\right)-F^*(y)\right| \leq$$
$$\left|F\left(y,t,\overline{A}_{[t-s,t]},f(s)\right)-E\left[F_t|f(s),G_\kappa\right]\right|+\left|E\left[F_t|f(s),G_\kappa\right]-E_sF_t\right|+\left|E_sF_t-F^*(y)\right|<\frac{\varepsilon}{8}+\frac{\varepsilon}{4}+\frac{\varepsilon}{8}=\frac{\varepsilon}{2}.$$

Using this, we have

$$\left\|\sup_y |F(y,t)-F^*(y)|\right\|_p^p =$$
$$\Pr[G_\kappa]\cdot E\left[\sup_y (F(y,t)-F^*(y))^p |G_\kappa\right]+(1-\Pr[G_\kappa])\cdot E\left[\sup_y (F(y,t)-F^*(y))^p \mid \text{not } G_\kappa\right]\leq$$
$$\left[\left(\frac{\varepsilon}{2}\right)^p+2^p\frac{\varepsilon}{8}\right]<2^p\varepsilon.$$

Since this is true for all $t > s$, it is also true for the lim sup. But ε was arbitrary, and the bound applies for all σ small enough given ε; hence the desired result follows.

The next technical lemma is used to bound the tails of the MA representations for both habits and commitments.

Lemma 5 *Let $g(u,s)$ be progressively measurable with respect to $\mathcal{F}_s$ satisfying $|g(u,s)| \leq K_1 e^{-K_2 u}$ for all u, s, and let*

$$G_t = \frac{1}{\overline{A}_t}\int_0^t g(t-s,s)\,\overline{A}_s dz_s.$$

For any $1 \leq p < \infty$, for σ_A small enough, there exists $M(p)$ such that $\|G_t\|_p \leq M(p)$.

Proof. We proceed by induction on t. Fix some $k > 0$. We show that (i) the desired bound holds when $t \leq k$, and (ii) if the bound holds for some t, it also holds for $t + k$. We begin by showing (ii), which is the more difficult part.

We can write

$$\|G_t\|_p \leq \left\|\frac{\overline{A}_{t-k}}{\overline{A}_t}\int_{t-k}^t g(t-s)\frac{\overline{A}_s}{\overline{A}_{u-k}}dz_s\right\|_p + \left\|\frac{\overline{A}_{t-k}}{\overline{A}_t}\right\|_p \cdot \left\|\frac{1}{\overline{A}_{t-k}}\int_0^{t-k} g(t-s)\,\overline{A}_s dz_s\right\|_p$$

where we used independence of the Brownian increments. Denoting $\overline{g}(u,s) = e^{K_2 k} g(u+k,s)$ we can rewrite the final term in brackets as

$$e^{-K_2 k}\cdot\frac{1}{\overline{A}_{t-k}}\int_0^{t-k}\overline{g}(t-k-s,s)\,A_s dz_s$$

where $|\overline{g}(u,s)| \leq K_1 e^{-K_2 u}$ by construction. By our induction assumption, this term has p-norm bounded by $e^{-K_2 k} \cdot M(p)$. To bound the remaining terms, first observe that by lognormality

$$\left\| \frac{\overline{A}_{t-k}}{\overline{A}_t} \right\|_p \leq K_p(\sigma_A, k)$$

for some $K_p(\sigma_A, k)$ that goes to one in σ_A for all k. Next note that

$$\left\| \frac{\overline{A}_{t-k}}{\overline{A}_t} \int_{t-k}^{t} g(t-s,s) \frac{\overline{A}_s}{\overline{A}_{t-k}} dz_s \right\|_p \leq \left\| \frac{\overline{A}_{t-k}}{\overline{A}_t} \right\|_{2p} \cdot \left\| \int_{t-k}^{t} g(t-s,s) \frac{\overline{A}_s}{\overline{A}_{t-k}} dz_s \right\|_{2p}$$

by the Cauchy-Schwarz inequality. Here

$$\left\| \frac{\overline{A}_{t-k}}{\overline{A}_t} \right\|_{2p} \leq K_{2p}(\sigma_A, k)$$

where $K_{2p}(\sigma_A, k)$ also goes to one in σ_A for all k. Finally, using standard bounds (e.,g., Karatzas and Shreve, 2008) for moments of the Ito integral, we obtain

$$\left\| \int_{t-k}^{t} g(t-s,s) \frac{\overline{A}_s}{\overline{A}_{t-k}} dz_s \right\|_{2p} \leq K_{2p} \left(\int_{t-k}^{t} K_1^2 \left\| \left(\frac{\overline{A}_s}{\overline{A}_{u-k}} \right)^2 \right\|_p ds \right)^{1/2}$$

which is bounded by $K_{2p} K_1 k \cdot K_{2p}(\sigma_A, k)$. Combining terms we obtain

$$\|G_t\|_p \leq K_{2p}^2(\sigma_A, k) \cdot K_{2p} K_1 k + K_p(\sigma_A, k) \cdot e^{-K_2 k} \cdot M(p).$$

It is easy to see that if

$$M(p) = \frac{K_{2p}^2(\sigma_A, k) \cdot K_{2p} K_1 k}{1 - K_p(\sigma_A, k) \cdot e^{-K_2 k}}$$

is positive, then the induction step follows. We can make sure that this is the case by first choosing some $k > 0$, and then picking $\overline{\sigma_A}$ small enough so that for all $\sigma_A \leq \overline{\sigma}_A$ we have $K_p(\sigma_A, k) < e^{K_2 k/2}$. With this choice of $M(p)$, the induction step follows; and (i) can be verified easily from the argument of the induction step.

We now present some results about habit models. We first show how to convert a habit representation with total consumption weights to one with A-weights, and to convert back.

Lemma 6 *Consider two habit models*

$$X_t = \int_0^t j(t-s)A_s ds + k(t)X_0$$

and

$$X_t = o(t)X_0 + \int_0^t \zeta(t-s)C_s ds$$

where the weight functions j, k, o and ζ are locally integrable. Then there is a one-to-one correspondence between these representations, and the weights are linked to each other through the Volterra integral equations

$$\zeta(u) = j(u) - \int_0^u \zeta(v)j(u-v)dv \tag{16}$$

$$o(t) = k(t) - \int_0^t \zeta(t-s)k(s)ds \tag{17}$$

with initial conditions $\zeta(0) = j(0)$, $o(0) = k(0)$. In particular, each C-average representation has a unique equivalent A-average representation.

Proof. Starting with the A-weighted habit model, consider the unique solution of the integral equations for ζ and o (see Lew, 1972 for existence and uniqueness) and define

$$\widetilde{X}_t = o(t)X_0 + \int_0^t \zeta(t-s)C_s ds.$$

We will show that $\widetilde{X}_t = X_t$ for all $t \geq 0$. First note that

$$\begin{aligned}
\widetilde{X}_t &= o(t)X_0 + \int_0^t \zeta(t-s)\left[A_s + X_s\right] ds \\
&= o(t)X_0 + \int_0^t \zeta(t-s)A_s + \zeta(t-s)\left[\int_0^s j(s-u)A_u du \; + k(s)X_0\right] ds \\
&= o(t)X_0 + \int_0^t A_s \left[\zeta(t-s) + \int_0^{t-s} j(u)\zeta(t-s-u)du\right] ds + X_0 \int_0^t \zeta(t-s)k(s)ds.
\end{aligned}$$

Equating coefficients, $X_t = \widetilde{X}_t$ holds if

$$j(t-s) = \zeta(t-s) + \int_0^{t-s} j(u)\zeta(t-s-u)\; du$$

or, with $t - s = u$,

$$\zeta(u) = j(u) - \int_0^u \zeta(v)j(u-v)dv$$

and

$$o(u) = k(u) - \int_0^u \zeta(u-v)k(v)dv.$$

Substituting in $u = 0$ gives $\zeta(0) = j(0)$ and $o(0) = k(0)$. The integral equation for $\zeta(u)$ then yields a unique solution, which can be used to determine $o(.)$. By the above argument, a pair of functions that solve these equations also give $X_t = \widetilde{X}_t$, which is the desired representation.

We next construct the best-fit habit model.

Lemma 7 *Let $\theta(u) = \xi^{*\prime}(u) \cdot e^{\mu_A u}$ and $\theta_0(u) = (\overline{x} - \xi^*(u)) \cdot e^{\mu_A u}$, then the habit model*

$$X_t^h = \int_0^t \theta(t-s) A_s ds + \theta_0(t) A_0 \tag{18}$$

generates the impulse response ξ^*.

Proof. Detrending both sides and integrating by parts (using that ξ^* is smooth)

$$\begin{aligned}\overline{X}_t^h &= \int_0^t \xi^{*\prime}(t-s)\overline{A}_s ds + [\overline{x} - \xi^*(t)] A_0 = \left[-\xi^*(t-u)\overline{A}_u\right]_0^t + \int_0^t \xi^*(t-s)\, d\overline{A}_s + [\overline{x} - \xi^*(t)] A_0 \\ &= \int_0^t \xi^*(t-s)\, d\overline{A}_s + \overline{x} A_0. \end{aligned} \tag{19}$$

Proof of Theorem 1. We are now ready to prove the main theorem of the section. We first focus on a sequence where $\sigma_A \to 0$. We can write

$$\frac{X_t - X_t^h}{\sigma_A A_t} = \frac{1}{\overline{A}_t} \int_0^t \left[\xi(t-s, f(s)) - \xi^*(t-s)\right] \overline{A}_s dz_s + \frac{E_0 \overline{X}_t - \overline{x}}{\overline{A}_t \sigma_A}.$$

We will bound the p-norm of this expression by breaking it into several pieces. Fix some $\varepsilon > 0$, let $k > 0$, and consider

$$\begin{aligned}&\left\| \frac{1}{\overline{A}_t} \int_0^{t-k} \left[\xi(t-s, f(s)) - \xi^*(t-s)\right] \overline{A}_s dz_s \right\|_p \\ &\qquad \leq \left\| \frac{\overline{A}_{t-k}}{\overline{A}_t} \right\|_{2p} \cdot \left\| \frac{1}{\overline{A}_{t-k}} \int_{t-k}^t \left[\xi(t-s, f(s)) - \xi^*(t-s)\right] \overline{A}_s dz_s \right\|_{2p} \\ &\qquad \leq K_{2p}(k, \sigma_A) \cdot M(2p) \cdot e^{-K_2 k}\end{aligned}$$

where we used Lemma 5. We can chose k large enough so that this entire term is less than $\varepsilon/3$.

Given this k, we next first bound

$$\begin{aligned}
&\left\|\frac{1}{\overline{A}_t}\int_{t-k}^{t}\left[\xi\left(t-s,f\left(s\right)\right)-\xi^*\left(t-s\right)\right]\overline{A}_s dz_s\right\|_p \\
&\qquad\leq \left\|\frac{\overline{A}_{t-k}}{\overline{A}_t}\right\|_{2p}\cdot\left\|\int_{t-k}^{t}\left[\xi\left(t-s,f\left(s\right)\right)-\xi^*\left(t-s\right)\right]\frac{\overline{A}_s}{\overline{A}_{t-k}}dz_s\right\|_{2p} \\
&\qquad\leq K_{2p}\left(k,\sigma_A\right)\cdot K_{2p}\left(k\right)\cdot\left[E\int_{t-k}^{t}\left[\xi\left(t-s,f\left(s\right)\right)-\xi^*\left(t-s\right)\right]^{2p}\left|\frac{\overline{A}_s}{\overline{A}_{t-k}}\right|^{2p}ds\right]^{1/2p} \\
&\qquad\leq K_{2p}\left(k,\sigma_A\right)\cdot K_{2p}\left(k\right)\cdot\left[E\int_{t-k}^{t}\left[\xi\left(t-s,f\left(s\right)\right)-\xi^*\left(t-s\right)\right]^{4p}ds\right]^{1/4p}\cdot\left[E\int_{t-k}^{t}\left|\frac{\overline{A}_s}{\overline{A}_{t-k}}\right|^{4p}ds\right]^{1/4p} \\
&\qquad\leq K_{2p}\left(k,\sigma_A\right)\cdot K_{2p}\left(k\right)\cdot K_{4p}\left(k,\sigma_A\right)\cdot\left[E\int_{t-k}^{t}\left[\xi\left(t-s,f\left(s\right)\right)-\xi^*\left(t-s\right)\right]^{4p}ds\right]^{1/4p}
\end{aligned}$$

where we repeatedly used the Cauchy-Schwarz inequality and a martingale moment bound, and where all constants are bounded as σ_A goes to zero. Now we use the idea that for σ_A small, ξ and ξ^* are close. Note that

$$\begin{aligned}
\xi\left(t-s,f\left(s\right)\right)-\xi^*\left(t-s\right) &= \int_L^U \xi\left(t-s,y\right)\cdot\left[f\left(t-s,y\right)-f^*\left(y\right)\right]dy \\
&= -\int_L^U \frac{\partial}{\partial y}\xi\left(t-s,y\right)\cdot\left[F\left(t-s,y\right)-F^*\left(y\right)\right]dy.
\end{aligned}$$

Here, for any fixed k, by Lemma 1, $\partial\xi\left(t-s,y\right)/\partial y$ is uniformly bounded in $\left(y,\sigma_A\right)\in[L,U]\times[0,\overline{\sigma}_A]$. Denoting this bound by $K\left(k\right)$, we have

$$E\left[\xi\left(t-s,f\left(s\right)\right)-\xi^*\left(t-s\right)\right]^{4p}<K^{4p}\left(k\right)\cdot E\sup_y\left|F\left(t-s,y\right)-F^*\left(y\right)\right|^{4p}.$$

Lemma 4 shows that the limsup over t of the last term goes to zero as $\sigma_A\to 0$. Thus given k and $\varepsilon>0$, for all σ_A small enough to make the entire term

$$\left\|\frac{1}{\overline{A}_t}\int_{t-k}^{t}\left[\xi\left(t-s,f\left(s\right)\right)-\xi^*\left(t-s\right)\right]\overline{A}_s dz_s\right\|_p<\frac{\varepsilon}{3}.$$

Finally, consider

$$\frac{1}{\sigma_A}\cdot\left\|\frac{E_0\overline{X}_t-\overline{x}}{\overline{A}_t}\right\|_p\leq\frac{1}{\sigma_A}\cdot\left\|\frac{1}{\overline{A}_t}\right\|_p\cdot K_1e^{-K_2t}\leq\frac{1}{\sigma_A}\cdot e^{K_3(p)\cdot\sigma_A^2t}\cdot K_1e^{-K_2t}.$$

If σ_A is small enough, then the limsup of this as $t\to\infty$ is zero. The result now follows for the case when $\sigma_A \to 0$.

We next consider a sequence where $\sigma_I \to \infty$. Here the key is to change the "clock," i.e., the speed with which we go through the Brownian sample paths. This effectively reduces both σ_I and σ_A at the same rate, converting our sequence of models into one where $\sigma_A \to 0$, where the previous result applies. The following Lemma summarizes the key step.

Lemma 8 *Fix $\tau > 0$, and let $\left(\widetilde{a}_t^i, \widetilde{x}_t^i\right)$ denote the optimal solution of a model with deep parameters $\tau \cdot \left(\rho, r, \pi, \sigma^2, \pi_I, \sigma_i^2\right)$, fixed costs $\overline{\lambda} = \left(\overline{\lambda}_1, \overline{\lambda}_2\right)$, curvature γ and relative preference κ. Then the process $\left(\widetilde{a}_t^i, \widetilde{x}_t^i\right)$ has the same distribution as $\tau \cdot \left(a_{\tau t}^i, x_{\tau t}^i\right)$: rescaling the time dimension acts the same way as rescaling the parameters of the model.*

Proof. We verify directly that changing the clock is equivalent to rescaling the relevant parameters in the setup of the problem. Maximizing the consumer's problem in the original model is equivalent to maximizing

$$E \int_0^\infty e^{-\rho t \tau} \left(\frac{a_{\tau t}^{1-\gamma}}{1-\gamma} + \mu \frac{x_{\tau t}^{1-\gamma}}{1-\gamma} \right) dt$$

which is proportional to the objective function in the model with new parameters. Similarly, the budget constraint of the original model implies

$$dw_{\tau t} = \left[\left(\tau r + \alpha_{\tau t} \tau \pi + \alpha_{\tau t}^i \tau \pi_I \right) w_t - \tau c_t \right] dt + \alpha_{\tau t} w_{\tau t} \sigma \tau^{1/2} dz_{\tau t} + \alpha_{\tau t}^i w_{\tau t} \sigma_i \tau^{1/2} dz_{\tau t}^i$$

on all non-adjustment dates due to the scaling invariance of Brownian motion. Finally, on adjustment dates, $dw = \overline{\lambda}_1 x_{t-}/r + \overline{\lambda}_2 x_t / r = \overline{\lambda}_1 \cdot \tau x_{t-} / (\tau r) + \overline{\lambda}_2 \cdot \tau x_t / (\tau r)$. Since the optimal policy is unique, the claim follows.

Now consider a sequence of models where $\sigma_I \to \infty$ and let $\tau = (\sigma_I)^{-2}$. According to the lemma, after changing the clock, dynamics will be identical to a model with parameters

$$\left(\tau \sigma_I^2, \tau \sigma_A^2, \tau r, \tau \mu_A, \gamma, \overline{\lambda}_1, \overline{\lambda}_2, \kappa \right) = \left(1, \tau \sigma_A^2, \tau r, \tau \mu_A, \overline{\lambda}_1, \overline{\lambda}_2, \kappa \right).$$

By construction, along this sequence aggregate risk goes to zero, while other parameters remain bounded as needed. Thus this model is very close to the equivalent habit representation; but then so is the original model where $\sigma_I \to \infty$, as desired.

Proofs for Section 5

Proof of Proposition 5. (1) Excess smoothness. This follows because X is of bounded variation and hence does not adjust instantaneously. (2) Excess sensitivity. If $s_1 \to 0$ and $s_2 \to \infty$ then almost all households adjust during $(t + s_1, t + s_2)$ and hence β_2 converges to one.

Proof of Proposition 6. (i) For the habit model, $E\left[\left|\log X^h_{t+\Delta t} - \log X^h_t\right| \mid B^{\Delta A}_{\Delta t}\right] \to 0$ by definition of the habit stock as a time average. With commitments, there exists $p > 0$ such that for any t, the probability of the following event is at least p: a mass of at least p in $F(t)$ is within distance Δ from L, and a mass of at least p is within distance ΔA from U. As $\Delta t \to 0$, the mass of people within ΔA of one of these boundaries all adjust, while the mass of adjusters on the other boundary tends to zero. It follows that for Δt small there exists $K > 0$ such that when $\Delta A_t > \Delta A$ we have $\Delta \log X_t > K$ and when $\Delta A_t < -\Delta A$ we have $\Delta \log X_t < -K$. The claim follows.

(ii) The existence of $K > 0$ from (i) immediately implies that $\beta^c_{1,ext}$ is bounded away from zero, while $\beta^c_1 \to 0$. In the habit model, we always have $X_t = \int_0^t \theta(t-s) A_s ds$ and hence $\beta^h_{1,ext}$ and β^h_1 both go to zero.

Proof of Proposition 7. (i) Our first goal is to compute the value function of the habit agent. Let ψ be defined so that the value function of the Merton consumption problem in the environment of the representative habit consumer, but without habit, is $\psi W^{1-\gamma}/(1-\gamma)$. By the envelope theorem, this Merton agent has consumption policy $c = \psi^{-1/\gamma} W$. The surplus consumption of our habit agent is identical to the consumption of a Merton agent, because they solve the same maximization problem. Hence, if the habit consumer sets his initial surplus consumption to be A_0, the dollar cost of his lifetime surplus consumption expenditure is $A_0 \psi^{1/\gamma}$.

To proceed, we now evaluate the lifetime budget constraint of the habit consumer. Each dollar of consumption spending in a period also creates future expenditure in the form of increased habit. Suppose $1 + B$ dollars is the present value of these future expenditures for a dollar of consumption spending today, where $B = 0$ with no habits. Then B must satisfy

$$B = \int_{u=0}^{\infty} \theta(u) e^{-ru} du \cdot (1 + B)$$

because each dollar of consumption creates $\theta(u)$ habit spending u periods ahead, which has a total cost of $\theta(u)(1 + B)$ in period u dollars, which we must then discount back at the riskfree rate because these payments are certain. Solving yields

$$B=\frac{1}{1-\int_{u=0}^{\infty}\theta\left(u\right)e^{-ru}du}.$$

At any time t, our habit consumer also has pre-existing habit created by his past consumption. The dollar value of the expenditures generated is

$$Z_t=\left(1+B\right)\cdot\left[\int_{s=0}^{t}C_{t-s}\int_{s}^{\infty}\theta\left(u\right)e^{-r(u-s)}du\ ds+\int_{s=t}^{\infty}\theta_0\left(u\right)X_0e^{-ru}du\right]$$

where the term in parenthesis measures future consumption expenditures created by habits established before t, discounted back at the riskfree rate because these are certain; and the factor $1+B$ is included because each dollar of consumption spending has this total expenditure cost.

The consumer's lifetime budget constraint must then satisfy

$$W_t=A_t\cdot\psi^{1/\gamma}\left(1+B\right)+Z_t$$

and his lifetime utility from surplus consumption, by the Merton value function, is simply $\psi^{1/\gamma}A_t^{1-\gamma}/\left(1-\gamma\right)$. Combining these equations yields

$$V_t^{habit}\left(W_t,X_t\right)=\frac{\psi}{1-\gamma}\left(\frac{W_t-Z_t}{1+B}\right)^{1-\gamma}.$$

The welfare of an individual commitment agent for a move-inducing negative wealth shock is proportional to $\left(w-\lambda_1 x\right)^{1-\gamma}/\left(1-\gamma\right)$.

Now compare the welfare cost of shocks in the commitment and the habit economies. As wealth falls to zero, if $Z_t>0$ then the marginal utility of the habit agent will be driven to infinity even with a finite shock. In contrast, when $\lambda_1=0$, the marginal utility of the commitment agent only blows up when all his wealth is taken. It follows that for large finite shocks, $\Pi\left(q,b\right)$ is higher for the habit agent than in the commitment economy.

(ii) The portfolio share of stocks for the habit agent is inversely related to his coefficient of relative risk aversion over wealth, which approaches infinity as W_t falls to Z_t. In contrast, the portfolio share of stocks in the commitment economy is bounded by a function of the highest possible relative risk aversion in the (S,s) band, which is a finite number.

Proof of Proposition 8. (i) In Given the assumptions, by continuity the (S,s) bands along this sequence will converge to the band of the limit economy. In Θ^*, agents in the interior of the band never adjust, hence $T_* (p|x_0) = \infty$. For n finite, agents does adjust eventually, but since the drift and variance of y goes to zero, the expected time to adjustment approaches infinity. In the habit model, x never changes, hence $T^{h,n} (p|x_0) = \infty$.

(ii) Begin with the commitment model. The agent in the limit economy never moves, and hence his value function is proportional to $(W - x/r)^{1-\gamma} / (1-\gamma)$. It follows that the coefficient of relative risk aversion $CRRA^* (W_0, x_0) = \gamma W_0 / (W_0 - x_0/r)$. Now consider an agent in economy n. Let p_0 denote the total dollar value at date zero of his total commitment expenditures on his current home. Given positive risk and growth, this agent does move eventually, implying $p_0 < x_0/r$. One policy available to this consumer at any wealth W is to maintain his spending and moving patterns on current commitments, and adjust spending proportionally on all other goods relative to the optimal policy with initial wealth W_0. Given that $\lambda_1 = 0$, this policy yields lifetime utility $V_n (W_0, x_0) (W - p_0)^{1-\gamma} / (W_0 - p_0)^{1-\gamma}$. This is a lower bound for the agent's true value function, and the both equal $V_n (W_0, x_0)$ at W_0. It follows that the lower bound has higher curvature at W_0. As a result, $CRRA^n (W_0, x_0) \leq \gamma W_0 / (W_0 - p_0)$. Since $p_0 < x_0/r$, we have $CRRA^n (W_0, x_0) < CRRA^* (W_0, x_0)$. Hence for b small, the Arrow-Pratt approximation implies $\Pi^n (q, b) < \Pi^* (q, b)$ uniformly in n.

In the habit model, the value function in every economy is proportional to $(W - x/r)^{1-\gamma} / (1-\gamma)$, and hence $\Pi^{h,n} (q, b) = \Pi^{h*} (q, b)$.

Appendix B: Simulations

Bellman equation and ODE characterization for the commitments model. In the simulations we use an ODE characterization of the optimal policy that builds on a similar characterization for the one-good model by Grossman and Laroque. To develop this ODE, we must study the Bellman equation of the commitment agent. Denote the value function by $V (W, x)$, then the Bellman equation between adjustment dates is

$$\rho V (W, x) = \max_{\alpha, \alpha} \left[\kappa \frac{a^{1-\gamma}}{1-\gamma} + \frac{x^{1-\gamma}}{1-\gamma} + V_1 (W, x) EdW + \frac{1}{2} V_{11} (W, x) Var (dW) \right].$$

Following Grossman and Laroque, let $y = W/X - \lambda_1$ and define $h(y) = x^{-1+\gamma} V(W,x) = V(W/x, 1)$. Dividing through by $x^{1-\gamma}$ in the Bellman equation we obtain

$$\rho h(y) = \max_{a,\alpha} \left[\kappa \frac{(a/x)^{1-\gamma}}{1-\gamma} + \frac{1}{1-\gamma} + h'(y) Edy + \frac{1}{2} h''(y) Var(dy) \right]$$

and the budget constraint yields

$$dy = ((y+\lambda_1)(r+\alpha\pi) - 1 - a/x) dt + (y+\lambda_1) \alpha\sigma dz.$$

Maximizing in α, the optimal portfolio satisfies

$$\alpha(y+\lambda_1) = \frac{-h'(y)}{h''(y)} \frac{\pi}{\sigma^2}$$

and adjustable consumption is

$$\frac{a}{x} = \left[\frac{h'(y)}{\kappa} \right]^{-1/\gamma}.$$

Substituting back into the Bellman equation we obtain

$$\rho h(y) = h'(y)^{1-1/\gamma} \kappa^{1/\gamma} \frac{\gamma}{1-\gamma} + \frac{1}{1-\gamma} + h'(y)\left[(y+\lambda_1) r - 1\right] - \frac{1}{2} \frac{h'(y)^2}{h''(y)} \frac{\pi^2}{\sigma^2}.$$

This is an ordinary differential equation for $h(y)$. To obtain boundary conditions, note that on an adjustment date the value function equals

$$\begin{aligned}
\frac{V(W,x)}{x^{1-\gamma}} &= \frac{1}{x^{1-\gamma}} \max_{x'} V\left(W - \lambda_1 x - \lambda_2 x', x'\right) \\
&= \left(\frac{W-\lambda_1 x}{x} \right)^{1-\gamma} \cdot \max_{x'} \left(\frac{x'}{W-\lambda_1 x} \right)^{1-\gamma} \cdot V\left(\frac{W-\lambda_1 x}{x'} - \lambda_2, 1 \right) \\
&= \left(\frac{W-\lambda_1 x}{x} \right)^{1-\gamma} \cdot \max_{y} (y+\lambda_1+\lambda_2)^{-1+\gamma} h(y).
\end{aligned}$$

Define

$$M = \max_y (y+\lambda_1+\lambda_2)^{-1+\gamma} h(y)$$

then by the above reasoning, at the edges of the inaction band, denoted y_1 and y_2 we have

$$h(y_i) = M y_i^{1-\gamma}$$

moreover, smooth pasting implies

$$h'(y_i) = M(1-\gamma) y_i^{-\gamma}.$$

Finally, the target value of y satisfies

$$y^* = \arg\max (y + \lambda_1 + \lambda_2)^{-1+\gamma} h(y).$$

To numerically solve the ODE subject to these conditions, we follow the approach outlined by Grossman and Laroque. We first pick some M, pick y_1, solve the ODE with initial conditions as given above. If there is no y_2 for which the boundary conditions are satisfied, then we start with a different y_1. If the boundary conditions do hold for some y_2, then we check if M satisfies the equation above; if not, we start with a different M.

Parameters used in simulations. Our general strategy is to choose deep parameters to generate variation in the key consumption risk parameters σ_I and σ_A while holding fixed consumption growth. In all four environments shown in Figures 1-3, the parameters $(\gamma, \kappa, \lambda_1, \lambda_2, \delta) = (2, 1, 1, 0, .0326)$ are held fixed. The other parameters and the implied values of consumption risk, individual and aggregate growth are shown in the table. Figure 4 uses the same parameters as (c) and (d) except that $\kappa = .01$, ensuring that habit is on average about 80% of consumption.

The Sharpe-ratio of the aggregate stock market is lower than in the data, while long-run aggregate consumption risk is higher, a variant of the equity premium puzzle.

	π_M/σ_M	π_E/σ_E	r	σ_A	σ_I	μ_a	μ_A
(a) High aggr, low idiosyncr risk	20%	10%	3.24%	10%	5%	1.24%	1.37%
(b) High aggr, high idiosyncr risk	20%	20%	1%	10%	10%	.87%	1.37%
(c) Low aggr, low idiosyncr risk	10%	10%	4.74%	5%	5%	1.24%	1.37%
(d) Low aggr, high idiosyncr risk	10%	20%	2.5%	5%	10%	.87%	1.37%

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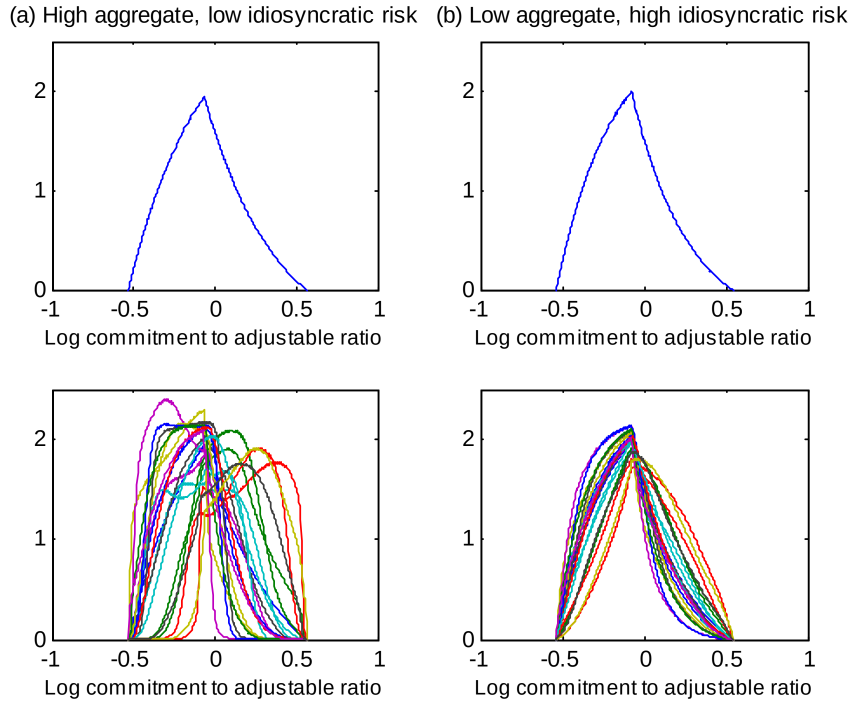
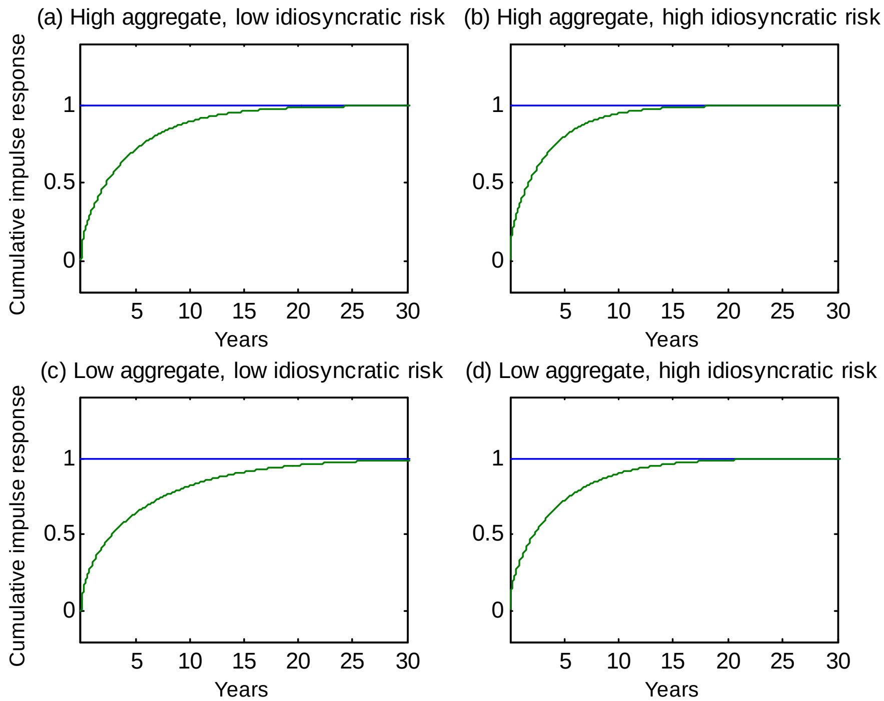
FIGURE 1: Cross-sectional consumption distributions



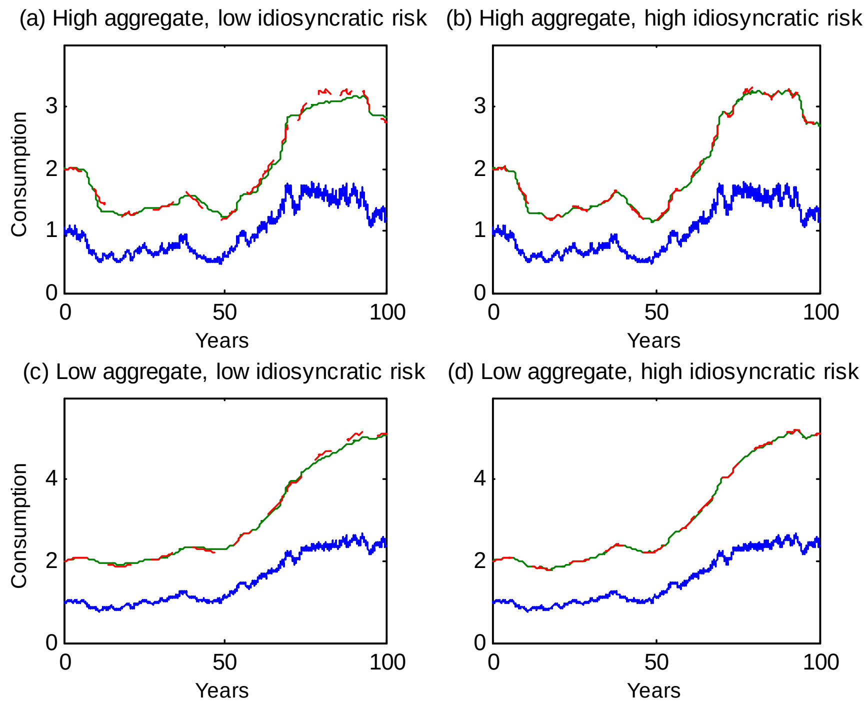
NOTE–Figures show cross-sectional densities of the log commitment to adjustables consumption ratio in two environments. For both environments, the top panel shows the long run steady state f^* while the bottom panel shows twenty realizations over a simulation corresponding to 100 years. Environment (a) has high aggregate risk ($\sigma_A = .1$) and low idiosyncratic risk ($\sigma_I = .05$) while environment (b) has low aggregate risk ($\sigma_A = .05$) and high idiosyncratic risk ($\sigma_I = .1$). See Appendix B for the parameters used in these simulations.

FIGURE 2: Normalized impulse response functions



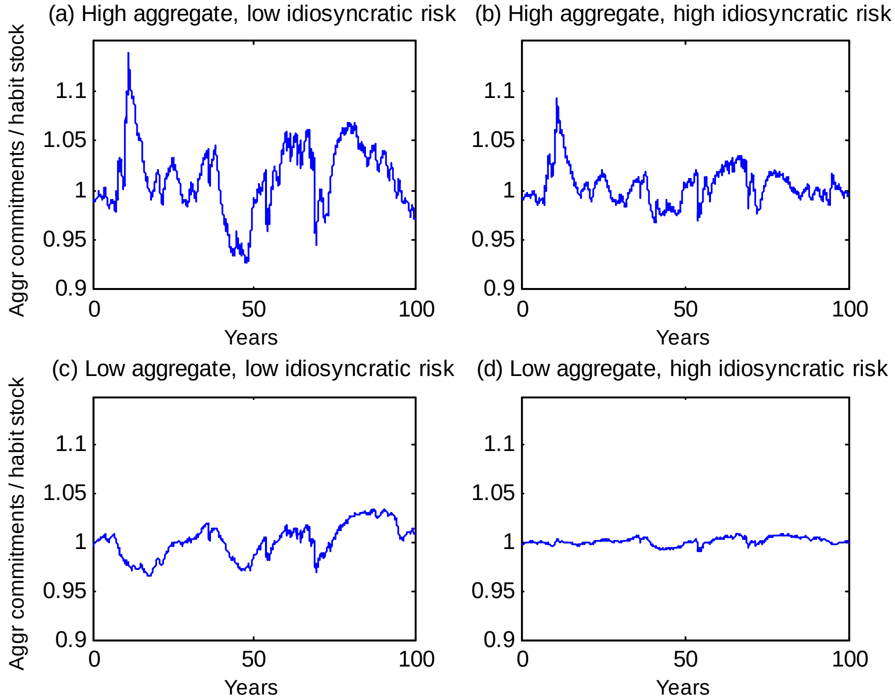
NOTE– Figures plot the normalized cumulative impulse response function of aggregate commitment consumption $\xi^*(t)/\overline{x}$ as a function of time elapsed after a shock in four environments with high (.1) and low (.05) aggregate and idiosyncratic risk. See Appendix B for the parameters used in these simulations.

FIGURE 3A: Aggregate dynamics of commitments and reduced form habit



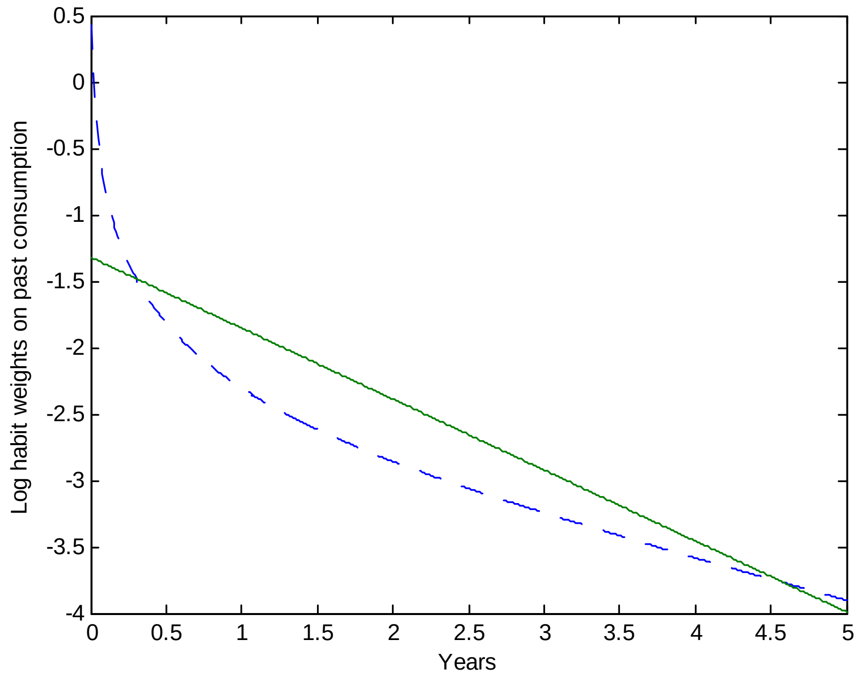
NOTE–Figures show commitments (green, solid) and reduced form habit (red, dashed) paths together with the evolution of permanent income (blue solid line in the bottom) in four environments with high (.1) and low (.05) aggregate and idiosyncratic risk. See Appendix B for the parameters used in these simulations.

FIGURE 3B: Ratio of aggregate commitments to fixed-weight habit stock



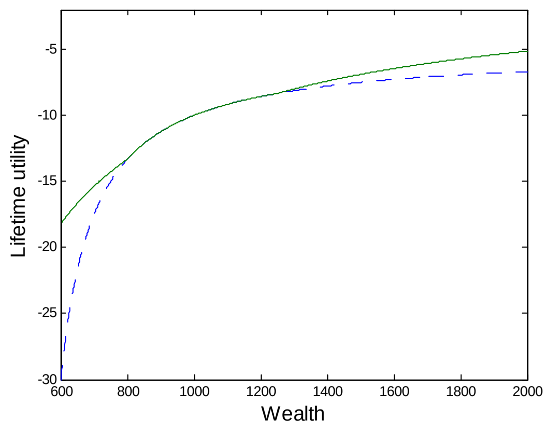
NOTE–Figures show the ratio of aggregate commitments and habit in the same four environments depicted in Figure 3a.

FIGURE 4: Log habit weights for exponential and commitment-based model



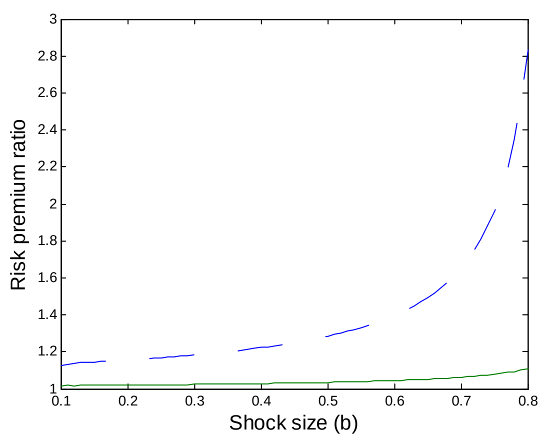
NOTE–Figure plots log consumption habit weights for the reduced-form exponential habit model (green solid line) and commitments model (blue dashed curve). Exponential habit parameters are from Table 1, column 5 of Constantinides (1990). Commitment model is the low aggregate, high idiosyncratic risk environment of Figure 3, with parameters given in Appendix B.

FIGURE 5: Value function in commitment and habit models with zero risk and growth



NOTE–Value as a function of wealth of a commitment agent (solid line) and the matching habit agent (dashed line) in an economy with zero consumption risk and no growth. The value function of the habit agent is shifted vertically to account for the utility value of commitments in the inaction region.

FIGURE 6: Ratio of risk premium in habit and commitments model vs. shock size



NOTE– Blue dashed line plots ratio of proportional risk premium of habit and commitment models as a function of shock size (b) in an environment with low idiosyncratic risk ($\sigma_I = .05$). Green solid line plots the same ratio with high idiosyncratic risk ($\sigma_I = .1$). The underlying risk is a negative shock realized with probability $q = 1\%$ that reduces wealth by a share b.

TABLE 1
Speed of adjustment of the commitment-based reference point

Aggregate risk	Idiosync risk	Riskfree rate	Individ cons growth	How many yrs till X adjusts p? (p=1 means full adjustment) p=0.25	p=0.5	p=0.75
A. Adjustment cost= 1* annual consumption						
10%	10%	1%	0.87%	0.44	1.73	4.24
5%	10%	2.50%	0.87%	0.55	2.24	5.6
10%	5%	2.50%	0.87%	0.6	2.34	5.73
5%	5%	4%	0.87%	0.84	3.4	8.64
10%	10%	4%	2.37%	0.4	1.63	4.15
B. Adjustment cost= 5* annual consumption						
10%	10%	1%	0.87%	1.06	4.15	10.26
5%	10%	2.50%	0.87%	1.15	4.84	12.62
10%	5%	2.50%	0.87%	1.46	5.67	13.79
10%	10%	4%	2.37%	0.73	3.1	8.39

NOTE-Table reports time until partial adjustment of the commitment based reference point is expected to occur in the aggregate economy. Top panel reports results when adjustment cost of commitments equals annual consumption (λ_1=1); bottom panel when adjustment cost is five times annual consumption value (λ_1=5). Consumption risk is varied by changing the Sharpe-ratio of idiosyncratic and aggregate investments (π/σ=.1 for low and .2 for high risk). Except in last row of each panel, riskfree rate is chosen to hold fixed individual consumption growth across specifications. In all rows, γ=2, κ=1, δ=.0326, λ_2=0. See Appendix B for details.