Fishing for Fools[*]

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Abstract

We show that common market settings tend to amplify rather than reduce the effect of behavioral biases on prices and other market outcomes. We study two common market mechanisms, auctions and fixed-price markets, and establish three results. First, agents with upward-biased valuations have an amplified effect on market outcomes because markets over-select them relative to their population share. Intuitively, markets "fish for fools." Second, auctions are often more efficient at "fishing" than fixed-price markets because a larger share of biased agents is required for prices to move in the fixed-price setting. Third, sellers respond to this difference and choose the less efficient but more profitable selling mechanism. They may also engage in inefficient complementary actions such as overproducing the good and over-recruiting buyers. We provide evidence from several markets, including eBay, housing markets, and financial markets.

Keywords: auctions, fixed-price markets, behavioral biases, overbidding, amplification

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1 Introduction

A common perception among economists is that market forces naturally limit the influence of behavioral biases on economic decision-making. In markets, high-stake incentives and repeated transactions discipline deviations (Stigler 1958; Becker 1957). Opportunities to learn from past mistakes and to sort into areas of specialization further reduce the impact of biases among market participants relative to the general population. The impact of any remaining behavioral participants is likely to be small because market outcomes are determined by the large mass of rational agents. These arguments suggest that behavioral biases may not have a large effect on prices and quantities, and on which market participant transacts with whom.

Some of the more recent empirical literature in behavioral economics challenges this perception and provides evidence of behavioral biases affecting outcomes in important markets, including high-stake and repeated decisions such as health-plan and other insurance choices, educational investment, and credit-card borrowing (Bhargava et al. 2017; Barseghyan et al. 2013; Bursztyn and Jensen 2015; Meier and Sprenger 2010). The evidence extends even to professional agents such a fund managers, central bankers, and CEOs (Puetz and Ruenzi 2011; Malmendier, Nagel, and Yan 2017; Malmendier and Tate 2005). Moreover, some of the literature has shown that it can be profitable for firms to target behavioral consumers (Grubb 2009, 2015; Hastings and Washington 2010; DellaVigna and Malmendier 2006; Eliaz and Spiegler 2006).

In this paper, we identify a new channel through which markets exacerbate rather than ameliorate the impact of biases: the ability of market mechanisms to "fish for fools." We argue that commonly-used selling mechanisms such as auctions or posted prices induce selection towards consumers with (more) biased beliefs. Key to this channel is heterogeneity in biases: Not everybody's decision making is distorted, and within the subset of biased agents, some are more biased than others. Intuitively, buyers who overestimate the value of a good are more likely to submit the highest bid in an auction, or enter a high-price market, and hence are more likely to transact than other, unbiased agents. Through this selection, biases can have oversized impacts on market outcomes.

One reason why this selection aspect has not featured prominently in previous analyses of "biases in markets" is the lack of consumer heterogeneity in many theoretical models. Much of the prior theoretical literature has focused on contrasting "the rational agent" with "the behavioral agent," assuming homogeneity within each type. Empirically, this approach leads to analyzing *average* behavior and testing for its consistency with the prediction of neoclassical economics.

In this paper, we argue that, by accounting for heterogeneity in biases, we uncover an important market force that works in the opposite direction of the presumed market-discipline effect: Market mechanisms help sellers "fish for fools."

First, we formalize the above intuition. We show that two commonly used market mechanisms, fixed-price markets and auctions, amplify rather than reduce the effect of behavioral biases. The reason is that these selling mechanisms induce *selection* in favor of buyers with biased beliefs. While a buyer who overestimates the value of an item is willing to overpay in any type of trade situation, also a non-market setting like a negotiation, markets increase the impact of biased beliefs on prices because biased buyers transact more frequently.

Second, we compare this amplification effect across different market settings. We show that, in auctions, even a small share of overbidding behavioral agents has a large effect on profits. The reason is that, by design, auctions select the highest bidder, and thus often a behavioral buyer, as winner. In fixed-price markets, "fishing for fools" tends to occur only at a higher share of behavioral agents, i. e., once the number of buyers willing to act on an inflated willingness to pay has exceeded a higher threshold.

Third, market forces further strengthen the amplification effect when sellers have a choice of market mechanism. Namely, sellers will choose the selling mechanism that amplifies the impact of behavioral types since they maximize their profits by exploiting these agents. For example, in a setting where fixed-price markets would be the efficient choice, sellers may nevertheless shift to auctions if only a small share of agents tend to overbid. Adding to this inefficiency, sellers may also overproduce the overvalued item and expend costs to attract additional buyers. As a result of these mechanisms, behavioral biases can have substantial influence on market outcomes and welfare.

In Section 2 we motivate the two key assumptions of our theoretical analysis—the presence of overbidding and of substantial heterogeneity in valuations and biases—with empirical evidence from various markets. We provide stylized facts and prior empirical findings from eBay, real-estate markets, and IPOs which show that a significant fraction of participants consistently overbid. These observations raise the question of whether the market mechanisms employed here help maximize profits, not only by identifying the bidder who values the good the most and is thus willing to pay the highest price, but also by identifying "over-bidders." At the same time, the micro-level data reveals that not every market participant bids above a rational upper bound. Instead, there is substantial heterogeneity and often only a small minority are "over-bidders."

In Section 3, we introduce a theoretical framework to illustrate how even a small share of over-bidders may exert a large influence on market outcomes. We first consider a simple market setting where a single seller sells one unit of a good to one of n agents. The intrinsic value v of the good is identical for all potential buyers, but with probability π an agent becomes a "fool," i. e., exerts upward bias, possibly due to bidding fever or rather factor inducing over-excitement about acquiring the item, and wrongly perceives the value to be $v_B > v$.[1] We consider two selling mechanisms, a second-price auction and a fixed-price market, and show that in both cases the market mechanism amplifies the effect of overestimation. That is, relative to a non-market setting such as a one-to-one negotiation between random agents, upward bias ("being a fool") has a stronger impact on prices and revenues. The amplification reflects that upward bias affects not only the willingness to pay in case of a transaction, but also the selection of buyers, i. e., the probability of entering a transaction. Both the auction and the market "fish for fools."

Second, we show that the selection effect increases revenues in the auction by more than in the fixed-price market for low probabilities π of bias. The reason is that the auction setting ensures a sale at a high (foolish) price as soon as there are "at least two fools." In the market, instead, a low π implies a high risk of no sale if the good is offered at a high price. In other words, while both the auction and the market fish for fools, the auction is more effective at fishing in that it allows the seller to take advantage of fools for lower values of π. The fixed-price market is a cruder tool that only allows for fishing when there are enough fools. This argument also implies that there is a range of π for which fools yield a higher profit in the auction.

We derive two further implications that reinforce the amplification result and point to its welfare

[1]To make our model broadly applicable, we model overvaluation generically and do not assume differential bias across markets.

implications. First, because of the efficiency of auctions in "fishing for fools" the seller has incentives to switch to the auction mechanism even when it is (cost) efficient to use the fixed-price mechanism. Second, a similar logic applies to the seller's other decisions, such as how much to produce and how much money to spend on attracting additional potential buyers. Both actions pay off even when they are inefficient as they allow the seller to better take advantage of the fishing effect. The key insight is that amplification gives the seller incentives to pursue inefficient actions.

Next, in Section 4, we present a richer model in which we allow for heterogeneous valuations of the good among the n agents. As before, there is heterogeneity in biases and agents become biased with independent probability π. In that case, they perceive their value of the good to be $v_B > v_H$, irrespective of their actual value.

All results from the simple model generalize to the richer model, albeit with some additional insights and subtleties. The revenue amplification effect remains essentially unchanged: "Fools" transact with a higher probability also under heterogeneous values. In addition, the richer setting allows us to show that the effect of biases on *allocations* is also amplified in both the auction and the fixed-price market: "Fools" win the object with amplified probability relative to their population share. Since this does not happen because of a high valuation, but because of their bias, their winnings can result in substantial misallocation and welfare loss.

The comparison between auction and fixed-price markets becomes more subtle with heterogeneous values. This is because, in the richer model, foolish misperception increases revenues via two channels, "demand shifting" and "price setting." The demand-shifting channel, which is only observed in the richer model, describes the effect of biased agents who are infra-marginal but whose presence still tilts the demand curve, which then results in a price increase. The price-setting channel, which is also present in the simple model, applies when fools are marginal and set the price (at v_B). We show that the auction continues to raise more revenue for a range of π, essentially because the price-setting channel is already active for low π in the auction, as two fools suffice to raise the price to v_B. In the fixed-price market, instead, the seller does not want to raise the price to v_B for the same low π. Thus only the the demand-shifting channel is active and its strength is merely proportional to π: a tilt in the demand of magnitude π will result in a price increase that is proportional to π. We also show that the results about sellers choosing the auction format "too often," producing "too much" of the good, and attracting "too many" potential buyers continue to hold in the richer setting.

Taken together, our theoretical results suggest that, due to market mechanisms overselecting them, a small share of biased agents can have large effects on prices, allocations, market choices, and welfare.

To support our model's predictions and gain insight into the magnitude of the effects, we present empirical results from online and offline auctions. We first turn to field evidence from the online auction site eBay. We study the unique case of board-game auctions for which a fixed-price listing was available simultaneously. Following Malmendier and Lee (2011), we say that a person overbids if he submits a bid above the simultaneously available fixed price. We find that only 17% of bidders overbid, but 43% of auctions conclude at a price that exceeds the fixed price. A simple calibration shows that this degree of amplification is fully consistent with our model. The same calibration also predicts that in about 73% of auctions the winner is biased, a finding consistent with our theoretical result on amplification in allocations, which suggests substantial misallocation. More broadly, we show evidence consistent with overbidding for several other products sold on eBay, suggesting that our theoretical predictions on amplification are relevant for a range of goods.

We then explore empirical evidence related to the other novel predictions of the model. Here, the existing evidence is suggestive, but indicates worthwhile paths for future data collection. For example, using data from both eBay and off-line auctions, we show that auctions are more common for unique items such as collectibles, and less common for commodity-type items. To the extent that biases are more prevalent in acquisitions of the former types of items, as implied by research on emotions affecting bidding in auctions (cf. Adam et al. 2011), these patterns are consistent with our prediction about sellers' choice of market mechanism. Another example is evidence from housing markets indicating that seller strategies to attract additional auction bidders are especially common in the presence of biased buyers, a pattern consistent with our prediction on attracting new buyers. These and other stylized facts indicate that both the basic amplification mechanism and its implications for allocations and welfare might be relevant in a broad set of markets.

Our work relates to several strands of literature. We build on theoretical work studying auctions and other selling mechanisms with rational agents. Reviews of theoretical and practical issues in auction design include Krishna (2010) and Klemperer (2002), among others. Most closely related is the research agenda emanating from Bulow and Klemperer (1996), that compares auctions with different types of negotiations, including Bulow and Klemperer (2009), Asker and Cantillon (2010), Bajari et al. (2001), and Bajari et al. (2008). This work demonstrates the benefits of auctions, but also those of other selling mechanisms under certain conditions, such as multidimensional heterogeneity or incomplete product design. We contribute to this work by exploring the impact of a small share of biased agents, and highlighting the new amplification mechanism.

Our approach and findings also relate to the behavioral mechanism design literature that studies other types of biased agents in auctions. Crawford and Iriberri (2007), for example, consider level-k agents as proposed by Nagel (1995), Stahl and Wilson (1994), Stahl and Wilson (1995). They find that agents following the random L1 rule tend to overbid in first-price auctions because they do not expect others to shade their bids; however, in contrast to our fools, these agents do not overbid in a second-price auction. Crawford, Kugler, Neeman, and Pauzner (2009) show that sellers prefer first-price auctions to second-price auctions when they believe buyers are predominantly following the L1 rule, similar in spirit to our result comparing second-price auctions and fixed price markets. The mechanism design literature also explores heterogeneous biases in other mechanisms. Saran (2011) studies the effect of naive traders, who report their true types without regard to incentives, in bilateral trading. He finds that the expropriation of naive traders' surplus to subsidize additional trading is a condition for constrained efficiency. Crawford (2019) finds similar results for level-k agents in bilateral trading. Our analysis provides parallels to this literature, and contributes by identifying and exploring the implications of a novel amplification mechanism.

Our paper also connects to the literatures on behavioral industrial organization and behavioral public economics. Much of this work, starting from DellaVigna and Malmendier (2004, 2006) and Eliaz and Spiegler (2006), and reviewed in Koszegi (2014) contrast the contract choice of sophisticated and naive agents. More closely related is the work on sellers' contract or product design when selling to both naive and sophisticated agents, which builds on Gabaix and Laibson (2006) and includes Heidhues, Kőszegi, and Murooka (2016), Heidhues and Kőszegi (2017), Schumacher (2016), and Heidhues and Koszegi (2018) among others.[2] Our emphasis on heterogeneity in biases is most closely tied to research in public economics documenting heterogeneity in attention and its implications for welfare and optimal taxation, including Taubinsky and Rees-Jones (2017) and Farhi and Gabaix (2019). Our contribution to this work is to identify and study the implications of the new amplification mechanism that emerges with heterogeneity in biases.

Finally, we build on experimental and field evidence documenting overbidding in auctions and markets, which we review in the next Section, where we motivate the key assumptions of our model.

2 Empirical Motivation

Our model is built on two key assumptions that differentiate our approach from much of the prior literature. First, we allow for buyers to overvalue goods and overbid. Second, we allow for heterogeneity in the existence and extent of the overvaluation bias. Before delving into the theory and intuition of how market mechanisms exacerbate, rather than ameliorate, the impact of biased behavior, we provide a selective overview of the empirical literature supporting both assumptions.

Overvaluation. The notion of overbidding in auctions is age-old. Already in ancient Rome, legal scholars debated whether a bid for public work provision was legally binding when the bidder suffered from "calor licitantis," bidder's heat (Malmendier 2002).[3] Much of the modern evidence comes from the laboratory. Several studies have found large and persistent overbidding in second-price auctions (e.g., Kagel and Levin 1993; Cooper and Fang 2008). There is also evidence on overbidding in laboratory ascending auctions, albeit smaller and less persistent (cf. Kagel, Harstad, and Levin 1987). Several studies explore the determinants and remedies of the laboratory overbidding.[4] In terms of field evidence, perhaps the most well-known and most studied example is

[2] Other models with heterogeneous biases include the studies of heterogeneity in naivete about time inconsistency by Eliaz and Spiegler (2006) and Grubb (2009).

[3] As argued by the Roman legal scholar Paulus, a winning bid for the right to collect taxes (which the Roman government farmed out) may not be valid if it has been "inflated beyond the usual sum due to bidding fever" (Corpus Iuris Civilis, D. 39,4,9 pr.).

[4] Kirchkamp, Poen, and Reiss (2009) find that outside options increase bidding in laboratory first-price auctions but have no impact on laboratory second-price auctions. Armantier and Treich (2009) attribute overbidding in first-price auctions to biased beliefs about the probability of winning. Kagel, Harstad, and Levin (1987) find that overbidding

overbidding in eBay auctions. Ever since eBay's founding and quick rise in popularity, studies and discussions of overbidding have abounded (Simonsohn and Ariely 2008; Malmendier and Lee 2011; Garratt, Walker, and Wooders 2012). In a particularly striking example, Jones (2011) shows that buyers systematically bid above the face value of Amazon gift cards.

Another market where overvaluation has been extensively documented is the market for initial public offerings. This market has both auction-like and fixed-price features as it combines book-building and price-setting. A widely studied empirical puzzle in this market is that, after an initial price jump during the first day of trading, issuing firms frequently underperform in the long run, suggesting that early buyers might be overbidding. For example, Ritter (1991) shows that, in the 1970s and 1980s, new issues underperformed a comparable set of listed stocks by 27.4 percent over the next three years. Similarly, Loughran and Ritter (1995) show that stocks issued in 1970-1990 have been poor long-run investments, both in initial public offering and in seasoned equity offering. In a review article, Ritter and Welch (2002) find that the average IPO underperforms the CRSP value-weighted market index by 23.4 percent, and underperforms seasoned companies with the same market capitalization and book-to-market ratio by 5.1 percent over the next three years. Similar patterns have been documented in several countries.[5] All of this evidence has been interpreted as an indication of initial buyers being too optimistic about the prospect of new shares and hence "overbidding" in the IPO.

A third market that features frequent discussions of overvaluation is real estate. This market also combines auction-like and fixed-price features. Several papers document evidence consistent with overexuberance and bubbles.[6] Most recently, Mian and Sufi (2019) argue that optimistic speculators played a key role in the 2002-2006 US housing boom, when recent home buyers had high expectations of house price growth, relative to the general population. An interesting example

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and auction revenue increase when bidders are given information about their opponents' values, while Goeree and Offerman (2002) document that increased information about the common value of a good reduces overbidding. Filiz-Ozbay and Ozbay (2007), building on Engelbrecht-Wiggans (1989), find evidence of overbidding to avoid regret from losing. Cooper and Fang (2008) identify both 'joy of winning' and 'spite' as motivators for overbidding.

[5] Ljungqvist (1997) finds that German IPOs lose more than 12 percent over their first three years of trading relative to the market. Levis (1993) shows that IPOs in the UK underperform a number of relevant benchmarks over three years. And Lee, Taylor, and Walter (1996) show that Australian IPOs significantly underperform market movements in the three-year period subsequent to listing.

[6] Cf. Stiglitz 1990; Kim and Suh 1993; Himmelberg et al. 2005; Hui and Yue 2006; Goodman and Thibodeau 2008; Glaeser et al. 2008; Yiu et al. 2013; Bourassa et al. 2016; Fabozzi and Xiao 2016.

comes from Ashenfelter and Genesove (1992) who examine the case of 83 units sold in a pooled condominium auction. When the sales of 31 units fell through afterwards, these units were resold in negotiated sales within a few weeks. The authors document that the negotiated sales occured at 87% of the original auction prices and the discount was highest for those units that were originally auctioned towards the beginning of the pooled auction, i. e., at the highest prices.

A final example is the market for corporate mergers, which is neither a pure auction nor a fixed-price market, but shares some common features with both. Roll (1986) and Malmendier and Tate (2005), among others, show that overconfident managers overpay for target companies and undertake value-destroying mergers. Based on this evidence from various contexts, we conclude that overvaluation plausibly exists across a wide range of markets.

Heterogeneity. In each of the markets discussed above, we also have evidence on heterogeneity in overvaluation.

For auctions, the above-mentioned study by Jones (2011) shows that 41% of closing bids exceeded the face value of the Amazon gift card, while the remaining 59% led to a price below the card's face value. The study also reveals significant variation in the extent (amount) of overbidding. Casari, Ham, and Kagel (2007) document heterogeneity in that women along with economics and business majors are more likely to overbid. Finally, in their study on the 'joy of winning' and 'spite' in overbidding, Cooper and Fang (2008) also show that, while some subjects closely follow rational predictions, others heavily deviate by paying for costly information about other bidders' valuations and overbid as a result of this information.

For IPOs, Chiang, Qian, and Sherman (2009) show that individual investors are more prone to overvaluation. In their analysis of Taiwanese IPOs, initial abnormal returns are significantly lower when the entry of individual investors into IPO auctions is high. Chiang, Hirshleifer, Qian, and Sherman (2011) complement these findings by showing that individual investors' returns steadily decrease as they gain more "experience," seemingly since they increasingly "overbid" after having experienced positive IPO returns. Thus there appears to be heterogeneity in overvaluation both by investor type and within investor.

In real estate markets, Mian and Sufi (2019) show that, in the period before the great recession, optimistic speculators represented less than 1.5% of the population but drove 40-70% of the increase in trading volume. Transactions of traditional home buyers with high credit scores, instead, declined in markets more exposed to speculation. This evidence suggests heterogeneity in overvaluation also in real-estate markets.

Finally, in the corporate merger market, Malmendier and Tate (2005) show that there is significant heterogeneity in overvaluation. Their widely used *Longholder* proxy identifies between 10% to 25% of CEOs as overconfident pre-2000, and a higher fraction in later sample periods. *Longholder* CEOs display significantly different investment, merger, financing behavior.

We conclude that there is robust evidence on heterogeneity in overvaluation across multiple markets; yet it does not feature in the existing theoretical frameworks. Our model in Sections 3 and 4 incorporates both overvaluation and heterogeneity in biases as key features. Consistent with the patterns described above, the model will show that even a small share of biased agents can have large effects on market outcomes, in addition to generating new predictions.

3 Simple Model

In this section, we develop a simple setting that allows us to identify the key forces emerging from the presence of biased agents in auctions and markets as well as heterogeneity in biased behavior across buyers. Specifically, we consider agents who, while engaged in the process of acquiring a good, overestimate its value and might overbid (and overpay) relative to their true willingness to pay ("fools"). Such a bias could reflect bidding fever, as in Ehrhart, Ott, and Abele (2015),[7] limited attention paid to alternative, lower-priced buying opportunities, as in Malmendier and Lee (2011), or other non-standard beliefs or cognitive limitations. We only consider upward bias, which has been the focus of the auction literature and, as documented in Section 2, is an empirically relevant phenomenon; however, we note that downward bias might also be relevant in some settings.

We start from a stylized model where the intrinsic value of the good to all buyers is identical. The only source of heterogeneity is whether a buyer is biased. We will show that selection amplifies

[7] See also Cooper and Fang (2008), Jones (2011), Heyman, Orhun, and Ariely (2004), and Ku, Malhotra, and Murnighan (2005).

the effect of overestimation in both auctions and fixed-price markets, that the impact of the bias is more prominent in auctions, and that sellers act on this difference by choosing the most-amplifying selling mechanism and inefficiently overproducing the good. In the next section, we will generalize these insights to a model with heterogeneity in intrinsic values, which also allows us to derive finer results regarding misallocation and hence welfare.

3.1 Setting

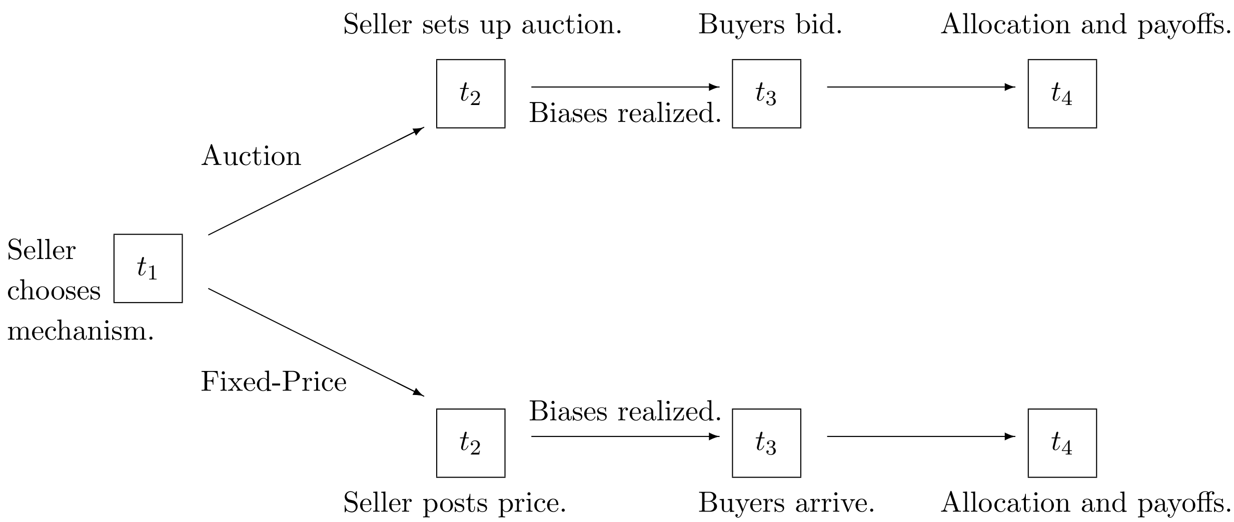
A single seller sells one unit of a good to one of n potential buyers. In our initial, simple setting, each buyer assigns the same value v to the good. With probability π, independently drawn across buyers, a buyer wrongly perceives the value of the good to be $v_B > v$. This bias affects the buyer's perception and willingness to pay while he is trying to acquire the item and he realizes his mistake afterwards. If he obtains the good, his consumption utility equals his true valuation v.

The seller has the choice to sell the good via an auction or at a fixed price. The auction is a second-price sealed-bid auction without a reservation value. We assume that bidders coordinate on the natural, (weakly) dominant "truth-telling equilibrium" and bid their perceived values. In the fixed-price market, the seller sets a price, and, if there is a buyer with valuation at or above the price, a sale is made. If there are multiple buyers with a high enough willingness to pay, one of them is chosen uniformly at random. We can think about this mechanism as a process where the seller sets the price, then buyers arrive at random times, and the first buyer with a sufficient willingness to pay obtains the good. The fixed cost of setting up an auction is c_a, and the fixed cost of setting up a fixed-price market is c_f. We usually assume $c_a > c_f$.

The timing of the game is illustrated in Figure 1. At t_1, the seller chooses the selling mechanism and pays its fixed cost. If she chooses the fixed-price market, she sets the price at t_2; else she sets up the auction. Biases are realized and buyers submit bids (in the auction) or express their willingness to buy (in the fixed-price market) at t_3. Finally, the good is allocated and all parties realize their payoffs at t_4. We are interested in the seller's optimal strategy in the perfect Bayesian equilibrium of this game when agents bid and are willing to pay their perceived values.

Discussion of modeling assumptions. Our model assumes that the potential bias affects buyers

Figure 1: **Timing of the Game**



in the auction and in the fixed-price market equally. We view this assumption to be a natural starting point. However, as the notion of "bidding fever" indicates, the occurrence of upward bias in buyers' willingness to pay might be more frequent in the auction. When this is the case, our results below understate the revenue benefit to sellers from using the auction format.

We also assume that the degree of misperception is the same for all biased buyers. We make this assumption for simplicity, to be able to starkly characterize the effect of biased bidders.

3.2 Results

We begin the analysis by characterizing expected revenue. In the second-price auction, all unbiased bidders submit bids of v, while all biased bidders bid v_B. Denoting by $q_k^n(\pi)$ the probability of at least k biased agents out of n people, the expected revenue of the seller in the auction is $v + q_2^n(\pi)(v_B - v)$. In the fixed-price market, the seller posts a price of either v or v_B. In the former case, the revenue is v, and in the latter case the expected revenue is $q_1^n(\pi)v_B = (1 - (1 - \pi)^n)v_B$. It is easy to see that there is a cutoff such that, when π is below the cutoff, the optimal price is v, and otherwise it is v_B.

To assess the magnitude of revenues under the different selling mechanisms, and their dependence on the probability of bias, it is helpful to consider a benchmark where the selection of the buyer is not influenced by whether that person is a fool. Specifically, suppose a randomly selected buyer meets up with the seller, and the two then split their (perceived) surplus in some fixed proportion, akin to a negotiation. With probability π the randomly selected buyer is biased, and the additional perceived surplus in that event is $v_B - v$. We dub the outcome of the negotiation with a randomly chosen buyer the *no-selection benchmark*. Under this benchmark, the presence of biased agents increases the seller's expected revenue by at most $\pi(v_B - v)$, and thus (at most) in proportion to the likelihood of bias π. As we will see, the result is different when agents are not only willing to pay more, but also more likely to obtain the product, if they are biased which is the case in both auctions and fixed-price markets.

Definition 1. We say that the revenue impact of biased buyers is *amplified relative to the no-selection benchmark* if it exceeds $\pi(v_B - v)$.

How do revenues in the auction and the fixed-price market compare to this benchmark? The following proposition provides the answer. In this and all future propositions, constants κ_1, κ_2, etc. do not depend on π or n, and thus remain fixed when we vary these parameters to examine comparative statics.[8]

Proposition 1. *The effect of biased buyers on expected revenue is amplified, relative to the no-selection benchmark,*

(a) in the auction for all π above a low bound (of order $1/n^2$), $\frac{\kappa_1}{(n-1)^2} \leq \pi < 1$, and

(b) in the fixed-price market for all π above a high bound (of order $1/n$), $\frac{\kappa_2}{n-1} \leq \pi < 1$.

The proofs of this and all subsequent results are in Appendix A.

Part (a) of Proposition 1 states that auctions amplify revenues when there are biased agents among the bidders as soon as the probability π of bias exceeds a low bound, $\frac{\kappa_1}{(n-1)^2}$. The key to understanding the amplification result is the insight that biases induce not only overbidding, but

[8] The parameters κ_i may depend on other model parameters, though, such as v or v_B. We give explicit formulas for the κ_i in the Appendix.

also selection: They influence who wins the auction and who sets the price. This force acts towards biased agents having an overproportional effect on prices relative to their frequency, or likelihood of bias, π, and thus relative to the no-selection benchmark. The auction "fishes for fools" in the sense that the influence of the bias on auction revenues exceeds the maximum influence the bias could have in a negotiation, $\pi(v_B - v)$, where biases affect buyers' willingness to pay, but not selection.

The bound in part (a), $\frac{\kappa_1}{(n-1)^2}$, is 'low' in the sense that it is of order $1/n^2$, i. e., goes to zero rapidly in n. For a more precise intuition, note that, as long as there are at least two biased agents, the two highest bids in a second-price auction come from biased agents, and hence the price will be v_B. The probability of at least two biased agents, in turn, is of order $\pi^2 n^2$ for small π.[9] This expression is proportional to n^2 and thus grows rapidly in n: Having two biased bidders in a slightly larger pool is much more likely. When π is of order $1/n^2$ or larger, the probability of having two biased agents exceeds π, and we get amplification.

Part (b) of the Proposition states that the fixed-price market also amplifies the revenue impact of biases when π exceeds a certain bound. The reason is that, when there are enough biased agents, it is optimal to set the fixed price at v_B and "start fishing." When this happens, the pricing generates selection, and hence amplifies revenues relative to the no-selection benchmark.

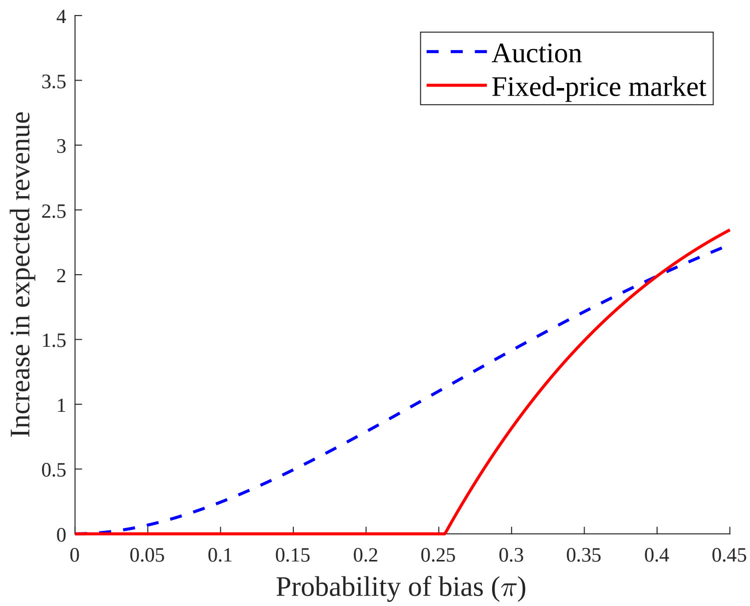
However, the threshold above which this selection is optimal, $\kappa_2/(n-1)$, is 'larger' in the fixed-price market in the sense that it is of order $1/n$. Intuitively, fishing for fools is harder under the fixed-price mechanism since setting the price at v_B may result in the costly outcome of no sale. For a more precise logic, note that for π small the probability of having at least one biased agent is of order $n\pi$.[10] Thus the fishing strategy generates expected profits of (approximately) $n\pi v_B$, while setting the price at v generates profits of v. Only for π above some constant times $1/n$ (where the constant is governed by the ratio v/v_B) the fishing strategy becomes more attractive.

So far, we have seen that distortions in valuation among some buyers affect prices and revenues, both because of such buyers' inflated willingness to pay and because of selection—biased agents

[9] To a second-order approximation in π, the probability of at least two biased agents is $\pi^2 \cdot n(n-1)/2$. Intuitively, the probability that two given agents are biased is π^2, there are $n(n-1)/2$ ways of choosing these two agents, and for π small the event in which more than two agents are biased can be ignored.

[10] Each of the n agents has the bias with probability π, and the event in which multiple agents have the bias can be ignored in a first-order approximation.

Figure 2: Impact of Bias on Expected Revenue in the Simple Model



Note: The figure calculates the expected revenue in the simple model from auctions and the fixed-price market, less the expected revenue when $\pi = 0$, for different values of π. The parameters used for this graph are $v = 10$, $v_B = 13$, and $n = 5$.

are more likely to acquire the item. The latter force amplifies the effect relative to a non-market, negotiation-like setting. Next we compare the revenue increase due to bias between the auction and the fixed-price market.

Proposition 2. *Biased buyers generate a larger revenue increase in the auction than in the fixed-price market for a range of low π, $0 < \pi \leq \frac{1}{\kappa_3 \cdot n}$.*

The proof builds on the intuitions described above. In the fixed-price market, for $\pi < 1/(\kappa_3 n)$, the fishing strategy is too risky and hence the seller sets the price at v. Thus, biased agents do not affect revenues, and the seller earns revenue v, for π in this range. In contrast, in the auction even a small positive π increases revenue. Intuitively, the auction is "more effective" at fishing even when π is small, while the fixed-price market is a cruder tool that only allows for fishing when there are enough "fools."

This result is illustrated in Figure 2, which plots the revenue impact of fools in both the auction and the fixed-price market as a function of π. The figure shows that expected revenue in the auction responds to the bias already at lower values of π, but eventually the bias impacts revenue in both environments. The figure also indicates that, for very high π, the amplification will be larger in the fixed-price market than in the auction. In this range of high π's, the probability that at least one agent is biased, and thus the revenue is v_B in the fixed-price market, is very high. The probability of at least two biased agents, and thus revenue v_B in the auction, instead, is less high.

Choice of mechanism. The revenue difference between the two selling mechanisms gives rise to an additional level of selection that further increases the impact of biases: the selection of the mechanism by the seller. Generally speaking, if different transactional formats, or forms of market organization, allow sellers to exploit the biases of buyers to a different extent, we would expect this difference to influence what format we see predominantly employed in transactions. In our context, if a seller can sell her product either in an auction or at a fixed price, which route will she choose, and how does her choice depend on the presence and extent of upward bias among buyers?

The optimality of a selling mechanism depends on its net revenues. Under our initial assumption that it is more costly to organize an auction than to offer the good at a fixed price, $c_a > c_f$, the fixed-price mechanism is socially more efficient than the auction: Both mechanisms allocate the good equally well, but organizing an auction is more expensive. In a world without distorted valuations, i. e., for $\pi = 0$, the seller chooses the socially efficient fixed-price mechanism as she earns revenue v under both mechanisms but only pays the lower cost c_f under the fixed-price mechanism.

The following corollary characterizes the seller's choice when $\pi > 0$, i. e., when buyers might overestimate the value of the good. Here, and for the rest of the section, we assume that $c_a - c_f$ is "sufficiently small" in the sense that it does not exceed a positive threshold, which is independent of π.[11] The upper bound ensures that neither the auction nor the fixed-price market is "too expensive" relative to the other, so that both choices can be optimal depending on the value of π.

Corollary 1. *The seller chooses the auction over the socially efficient fixed-price market for a range of values $\pi > 0$ and $c_a - c_f$ sufficiently small.*

[11] The precise specification of the bound is in Appendix A.

Corollary 1 is a direct consequence of Proposition 2: Since the auction is better at fishing, the seller decides to use it for some levels of $\pi > 0$, despite its higher fixed cost. This switch to auctions is inefficient; the lower-cost fixed price mechanism would have allocated the good just as efficiently.

Production and Marketing. The amplified revenue of the auction also incentivizes inefficient actions that complement selling the good. Here, we explore two examples: production and marketing.

First, consider the production decision. Suppose the cost of producing the good is k_p, drawn from some distribution with full support on $k_p \geq 0$. After observing the realization of k_p the seller decides whether to produce, and if she does, the game unfolds as before. If she chooses not to produce, the game ends and all parties get zero payoffs. Since $c_f < c_a$, efficiency dictates that the good is produced only if $k_p \leq v - c_f$. We say that the seller *overproduces* the good relative to the welfare-maximizing outcome if there is a $k_p > v - c_f$ for which she chooses to produce.

Biased buyers can distort the incentives of the seller, as shown by the following result.

Corollary 2. *There is a range of $\pi > 0$ for which the seller would produce efficiently under the fixed-price mechanism, but chooses the auction and overproduces.*

The result identifies a new force through which the presence of biased agents reduces social welfare as it alters the seller's choice of market mechanism. Biased bidders make the auction "too profitable" and hence induce the seller to overproduce the good.

Next, consider a complementary marketing decision. Suppose that the seller can attract one additional potential buyer if she expends a cost k_b. Note that, in the current setting, recruiting another buyer is socially inefficient as all buyers value the good the same ex ante. Indeed, for $\pi = 0$ the seller chooses the fixed-price mechanism and does not recruit.

Corollary 3. *There is a range of $\pi > 0$ for which the seller would not incur costs to attract additional buyers under the fixed-price mechanism, but chooses the auction and inefficiently incurs the cost to attract additional buyers, as long as k_b sufficiently small (given n).*

This result is based on the observation that the fishing effect arises "earlier," i. e., for lower π, in the auction than in the fixed-price market. The observation implies both that the seller prefers the auction as it generates higher revenue and that she has a stronger preference to attract additional interested buyers to the auction in order to further take advantage of the fishing effect. Note that we need k_b to be small enough given n because the value of one additional buyer becomes arbitrarily small as n grows.

In summary, the simple model with heterogeneous biases, but homogeneity in (true) valuations, illustrates how distortions in buyers' perception of the value of a good can interact with market forces to exacerbate the impact of the bias: Not only will biased agents tend to overpay, they are also particularly likely to transact with the seller. Sellers anticipate these effects and fine-tune their selection of a selling mechanism as well as their production and marketing to leverage the amplification effect.

4 Full Model with Heterogeneous Values

We next consider a more general framework where buyers' intrinsic values of the good are heterogeneous. This additional layer of heterogeneity sheds light on the intensive-margin effect of biases: The presence of biased agents can change the identity of the marginal *unbiased* agent who sets the price and who will have a higher valuation in the presence of biased agents. Moreover, because of this new channel, the bias increases revenue already for smaller frequencies $\pi > 0$, not just in the auction but also in the fixed-price market. This intensive-margin effect thus improves the "fishing for fools" in the fixed-price market already for small $\pi > 0$. Nevertheless, as we show below, the results of the previous section continue to hold qualitatively, including the stronger amplification in the auction, because the "extensive-margin" effect identified earlier is stronger than the new intensive-margin effect.

4.1 Setting

Assume that buyers' valuations of the good are distributed i. i. d. according to $F(v)$, which has support on $[v_L, v_H]$. As before, each of the n buyers becomes biased with independent probability π, and in that case perceives the value of the good during the auction to be $v_B > v_H$, irrespective of his true value v.[12] To simplify the analysis we make the following assumption about F:

Assumption 1. (a) $F(v)$ has density $f(v)$, and for some $0 < \underline{f} < \bar{f}$ it satisfies $\underline{f} \leq f(v) \leq \bar{f}$ $\forall v \in [v_L, v_H]$. (b) In addition $tf(t)$ is increasing for $t \in [v_L, v_H]$.

Part (a) of the assumption ensures that F has a well-behaved density. Part (b) is a regularity condition which guarantees that the seller's objective function in the fixed-price market is concave in the price. That condition is related to Myerson's regularity condition, versions of which are often used in auction theory to ensure that the seller's marginal revenue curve is downward sloping (Bulow and Klemperer 1996) and which, in our setting, would imply that the seller's problem is quasi-concave when $n = 1$ and $\pi = 0$. Our condition ensures concavity for all n and π. Assumption 1 holds for example when F is uniform on $[v_L, v_H]$.

As before, we compare a sealed-bid second-price auction and a fixed-price market. In the auction we continue to focus on the equilibrium in which agents bid their perceived values. In the market, the seller sets a fixed price p which, in the optimum, is either v_B (which would yield expected revenue $v_B(1-(1-\pi)^n)$) or the maximizer of

$$p \cdot (1 - (1-\pi)^n F(p)^n) \tag{1}$$

over $[v_L, v_H]$, where $(1-\pi)^n F(p)^n$ is the probability that there are no biased buyers and that all buyers' values are below p.

4.2 Results

We structure our results as in Section 3, and first establish amplification.

As before, we would like to interpret the magnitude of the revenue impact relative to a benchmark where the selection of the buyer is not influenced by the bias. Mirroring our approach in the simple model, we consider a situation where a buyer meets up with the seller, the buyer becomes biased with probability π, and the two split the *perceived* surplus in some fixed proportion, akin to

[12] For simplicity, we continue to model biased agents as having identical perceived values v_B. The results are qualitatively unaffected if we draw perceived values from a small range around v_B.

a negotiation. As before the additional perceived surplus from bias is $v_B - v$, and the impact on the seller's expected revenue thus at most $\pi(v_B - v)$. Since the additional surplus now varies with v, there are several possible no-selection benchmarks, such as $\pi(v_B - E[v])$ or $\pi(v_B - v_L)$. To err on the conservative side, we allow for a maximum non-amplification benchmark of $\pi(v_B - v_L)$.

Definition 2. In the model with heterogeneous $v \sim F[v_L, v_H]$, we say that the revenue impact of biased buyers is *amplified* relative to the no-selection benchmark if it exceeds $\pi(v_B - v_L)$.

How do auction and fixed-price revenues compare to this benchmark? The following propositions provide the answer. Similar to our notation before, constants $\tilde{\kappa_1}$, $\tilde{\kappa_2}$, etc. do not depend on π or n and thus remain fixed when we vary these parameters for comparative statics.[13]

Proposition 3. *The effect of biased buyers on expected revenue is amplified, relative to the no-selection benchmark, both*

(a) in the auction, for all $\frac{\tilde{\kappa}_1}{(n-1)^2} \leq \pi \leq \frac{1}{\tilde{\kappa}_1}$ *, and*

(b) in the fixed-price market, for all $\frac{\tilde{\kappa}_2}{n-1} \leq \pi \leq \frac{1}{\tilde{\kappa}_2}$.

Proposition 3 establishes amplification in both the auction and the fixed-price market. The result and the proof are analogous to Proposition 1. Part (a) states that the lower bound for amplification in the auction goes to zero at a fast quadratic rate in n, implying that the auction generates amplification for relatively low values of π. Part (b) establishes that the lower bound for amplification in the fixed-price market is relatively large, of order $1/n$, and the intuition parallels that of the earlier result: Only when there are enough biased agents is it optimal to set the price at v_B and "start fishing."

The main difference relative to Proposition 1 is that the constants $\tilde{\kappa}_1$ and $\tilde{\kappa}_2$ are new. The difference reflects that we are using an even more conservative (i. e., higher) no-selection benchmark in the setting with heterogeneous values than in the setting with homogeneous values, and thus meeting it requires different bounds on π. For the same reason, Proposition 3 requires an upper bound on π in both parts of the Proposition. The upper bound ensures that there is enough

[13] The $\tilde{\kappa}_i$ may depend on other model parameters such as v or v_B.

"room" for amplification. For example, when π is close to one, amplification beyond the no-selection benchmark of $\pi(v_B - v_L)$ is not feasible as the no-selection bound is strictly larger than the potential gain from biased agents in either mechanism. (Assumption 1 ensures that absent bias revenue is strictly above v_L because the density is positive anywhere in $[v_L, v_H]$.)

In the richer environment of this section, we can also discuss the implications of the bias for allocational efficiency and show that bias can have oversized effects. To formalize the basic idea in simple terms, we introduce the notion that biased buyers have an *amplified effect on allocations* if the probability that a biased buyer gets the good exceeds their expected population share π. Amplified allocation effects, in turn, can lead to large welfare effects: When a biased agent gets the good, his expected valuation is only $E[v]$, while absent bias the allocational welfare of both the auction and the market asymptotes to $v_H > E[v]$ as n grows.

Corollary 4. *The effect of biased buyers on allocations is amplified both in the auction and in the fixed-price market under the conditions stated in Proposition 3.*

The intuition is straightforward. In the auction, revenue amplification emerges because the probability of at least two biased agents exceeds π and, in that case, the person winning the good must be biased. In the fixed-price market, revenue amplification, i. e., a revenue gain exceeding $\pi(v_B - v_L)$, emerges when the seller sets the price at v_B which implies that a biased agent must get the good with probability exceeding π. In summary, the heterogenous-values model reveals that biases can have oversize effects not only on revenue but also on allocational efficiency.

The next result shows that the auction remains more effective in exploiting biased agents for a range of π also in the current setting.

Proposition 4. *Biased buyers generate a larger revenue increase in the auction than in the fixed-price market for a range of low* π, $\frac{\tilde{\kappa}_3}{(n-1)^2} \leq \pi \leq \frac{1}{\tilde{\kappa}_3 \cdot n}$.

The Proposition is analogous to Proposition 2 in the previous section, with the key novelty that, now, the interval for the range of π where the auction dominates the fixed-price market has a positive lower bound.[14] To understand this result, it is useful to distinguish between two channels

[14] As a result of this positive bound, the interval for π specified in the Proposition may be empty for n low, but will be non-empty for n sufficiently large.

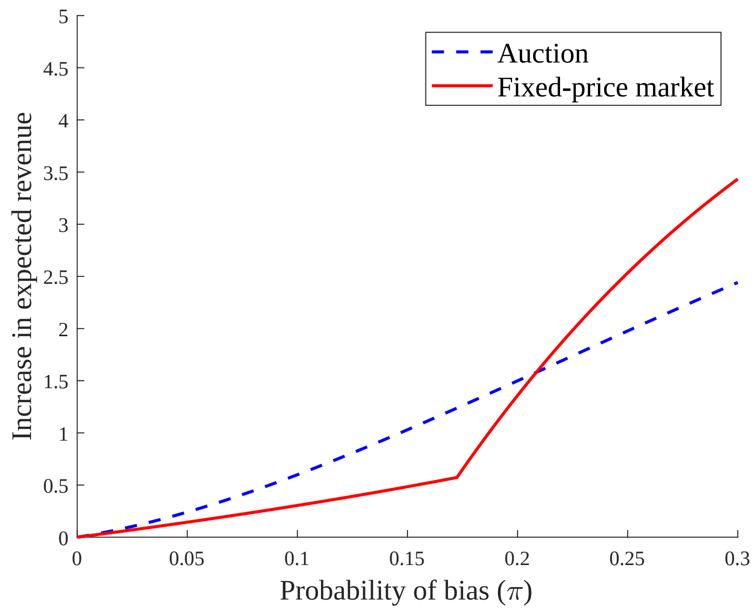
through which biases impact revenues in a setting with heterogeneous values: price setting and demand shifting. Biased valuations affect revenues via the price-setting channel when the marginal agent is biased, and hence the price is v_B. Biased valuations affect revenues via the demand-shifting channel when the biased agents are infra-marginal and hence do not directly set the price. In that case, their presence still shifts the demand curve and generates a price increase.

The second channel is absent in the homogenous-values model of the previous section because $F(v)$, and hence the demand curve, is degenerate. But it is present in the current heterogeneous-values model, and the reason of the positive lower bound for π. More specifically, the range of π specified in Proposition 4 implies that the seller does not find it optimal to set the price equal to v_B in the fixed-price market. Even so—and unlike the case of the simple model—biased agents still generate a revenue increase because their presence encourages the seller to increase p within the range $[v_L, v_H]$. This is the "demand-shifting" effect: Although biased agents do not set the price ($p < v_B$), their presence changes the identity of the marginal agent (the buyer with $v = p$), who sets the price. A higher-value unbiased agent becomes marginal and thus acts to increase revenue.

The key part of the proof is to show that in spite of the demand-shifting effect, the revenue gain from the bias is higher in the auction than in the market (over a range of π). To see the logic for this, consider the seller's problem, and suppose that π is increased from zero to some positive level within the range specified in Proposition 4. By the logic of the envelope theorem, the first-order revenue impact of this increase in π comes from the increase in the probability of a sale. (The effect coming from a change in the optimal price is second-order). Such an increase arises from the event in which (i) one buyer becomes biased while (ii) at $\pi = 0$ no potential buyer was interested in buying. The probability of (i) is proportional to $n\pi$ and thus quite high; the probability of (ii) is of order $1/n$ and thus quite low because, at $\pi = 0$, the seller sets the price such that the probability of no sale, a costly outcome, is low. The product of these terms is bounded by a constant times π. In contrast, the revenue gain in the auction can be made higher than any constant times π for an appropriate $\tilde{\kappa}_3$, similar to the proof of Proposition 1(a).

Figure 3 illustrates these effects by plotting the (total) revenue gains in the auction and in the fixed-price market as a function of π. Here, revenues respond even to a small share of biased agents

Figure 3: Impact of Bias on Expected Revenue in the Full Model



Note: The figure calculates the expected revenue in the simple model from auctions and the fixed-price market, less the expected revenue when $\pi = 0$, for different values of π. We assume $F(v)$ is uniform on $[v_L, v_H]$. The parameters used for this graph are $v_L = 5$, $v_H = 10$, $v_B = 13$, and $n = 5$.

in both mechanisms. This is due to the demand-shifting effect. But, when π is small, the response is larger in the auction since the auction activates the price-setting channel even for low bias. As π increases, the price-setting channel becomes active in the fixed-price market as well, and the bias-induced revenue increase in the fixed-price market ultimately exceeds that in the auction.

We now turn to explore the implications of these results for market choice, complementary actions, and efficiency. Because of the additional richness of the heterogeneous-values model, these results are easier to establish for n sufficiently large, by which we mean that there is some $\bar{n}$, which does not depend on π (but may depend on $F(.)$ and v_B), such that the statement holds for all $n \geq \bar{n}$. We also bound the extra cost of holding an auction compared to a fixed-price market, $c_a - c_f$ both from above and below. That is, implicit in all of the following corollaries is the statement that there exist $c_1 < c_2$, independent of n and π, such that the corollaries hold when $c_1 < c_a - c_f < c_2$.

Choice of mechanism. For market choice, we have the following result:

Corollary 5. *Let $c_1 < c_a - c_f < c_2$ and n be sufficiently large. Then the seller chooses the auction over the socially efficient fixed-price market for a range of $\pi > 0$.*

Thus, even in a world with heterogeneous values, the presence of fools can encourage sellers to shift from a fixed-price to an auction mechanism. This result follows directly from Proposition 4: Since the profit gain from biased buyers is higher in the auction for a range of low π's, sellers have a stronger incentive to use it.

Production and Marketing. Now consider the implications for production. As before, suppose that producing the good has a cost k_p for the seller, which is drawn from some distribution with full support on $k_p \geq 0$. In the setting with heterogeneous values, there are two new forces influencing welfare. First, even absent bias, buyers earn an information rent due to the heterogeneity in their valuations. Since the seller does not appropriate this rent, her revenue does not fully reflect the value of the good to the highest bidder, and hence she underproduces relative to the first best. Second, as we discussed in Corollary 4, bias results in misallocation when a low-value but biased agent wins the good instead of a high-value unbiased agent. Our assumption that n is sufficiently large ensures that the information rent is small, absent any bias, and both mechanisms generate revenue approaching v_H. This effectively shuts down the first force. Moreover, here we do not focus on the second force, i. e., misallocation, but the implications for inefficient production.

Production is efficient when the cost of production and selling is smaller than the expected highest value of the good among all buyers. Denoting by F_1^n the expected highest value out of n draws from distribution F, we say the production decision is efficient when the good is produced if and only if $k_p < F_1^n - c_f$. For any $\varepsilon > 0$, we define the production decision to be ε-efficient if the gap between the efficient and the actual cutoff is less than ε. We say that the seller *ε-overproduces* if she chooses a cutoff more than ε higher than efficient cutoff, and *ε-underproduces* if she chooses a cutoff more than ε lower than the efficient cutoff.

Corollary 6. *Let $c_1 < c_a - c_f < c_2$, $\varepsilon > 0$ be sufficiently small, and n be sufficiently large. There is a range of $\pi > 0$ for which the seller would produce ε-efficiently under the fixed-price mechanism, but chooses the auction and ε-overproduces.*

The corollary mirrors Corollary 2 from the simple setting with homogeneous values. A slight modification is that, because of the heterogeneity in values, even under the fixed-price mechanism the seller would produce only "approximately" efficiently.

Finally, suppose that at a cost k_b the seller can recruit one additional buyer.

Corollary 7. *Let $c_1 < c_a - c_f < c_2$ and n be sufficiently large. Then, for k_b sufficiently small, there is a range of $\pi > 0$ for which in the seller would not recruit under the fixed-price mechanism, but chooses the auction mechanism and inefficiently recruits.*

This result parallels that of Corollary 3 for our simple model and, similar to Corollary 3, follows from the higher marginal value of an additional buyer in the auction mechanism at low values of π. The result implies that recruiting additional buyers can be another potential channel which leads to inefficient outcomes in the auction market when a small fraction of buyers over-estimates the value of the good.

In summary, the presence of heterogeneous values does not alter the basic insight about amplification in auctions and fixed-price markets or the lesson that auctions have an advantage in fishing for fools for a range of low π. Moreover, with heterogeneous values fools also induce amplification in allocations which, in turn, generates misallocation. These insights also translate into predictions about complementary decisions regarding market choice, production, and marketing: They are distorted when biases significantly affect revenues, and auctions are effective in exploiting bias in such a revenue-enhancing fashion.

5 Overbidding, Amplification, and Market Choice: Evidence

How relevant are our theoretical findings for auction markets in practice? In Section 2, we provided several pieces of evidence for the distinctive assumptions in our model, overbidding and heterogeneity in biases. In Sections 3 and 4, we formalized the implications of these assumptions, which included results on the amplification of biases due to selection, the relative strength of the selection effect on revenue in fixed-price and auction markets, and on misallocation. In this section, we provide new evidence from eBay and present stylized facts from online and offline auctions consistent with our model predictions.

5.1 Field Evidence from eBay

Our analysis utilizes the data from Malmendier and Lee (2011). We first introduce the data set and discuss the findings first presented in Malmendier and Lee (2011) that are relevant to our model assumptions. We then derive further results that speak to our novel model predictions.

The core data set of Malmendier and Lee (2011) contains all eBay auctions of a popular educational board game, Cashflow 101, during a seven month period in 2004.[15] The key to the identification of overbidding in this setting is the simultaneously available fixed-price market on the auction website.[16] During the sample period, two retailers continuously sold brand new Cashflow 101 games on eBay at a fixed (buy-it-now, or BIN) price of \$129.95, and their listings were shown together with the auctions on the regular output screen for Cashflow 101.[17] Given the continuous availability of the game at this fixed price, rational eBay participants should never submit bids exceeding the fixed price. Thus, the number of agents who bid above the simultaneous fixed price is a conservative measure of overbidding. It excludes behavioral buyers who bid above their value but below the fixed price.

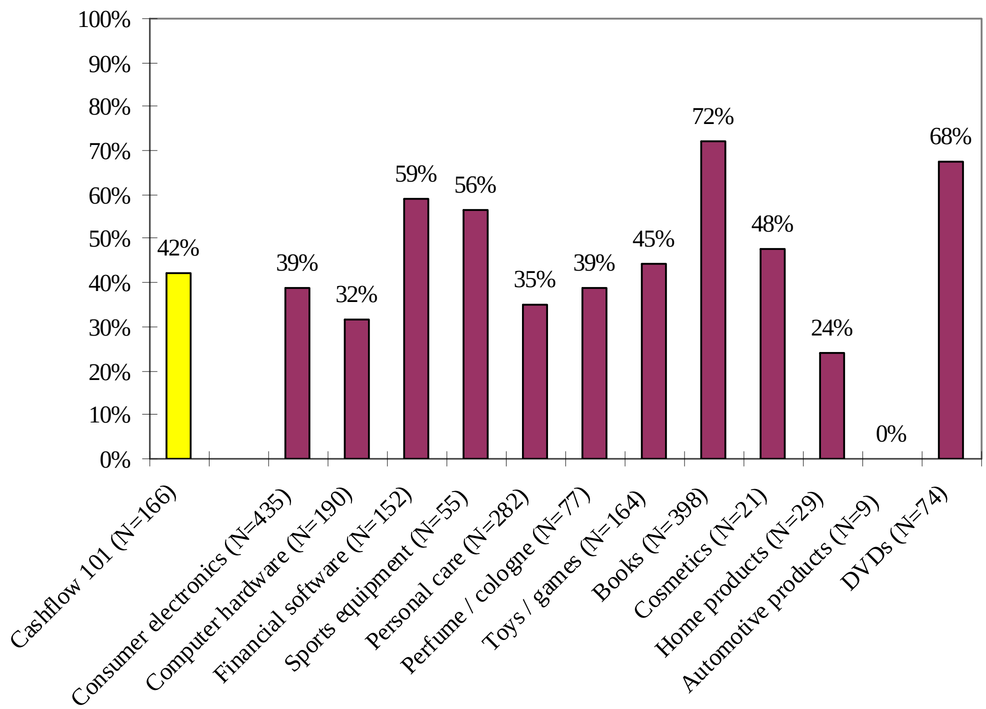
Table 1 provides summary statistics of these data. The sample consists of 166 listings with $2,353$ bids by 807 different bidders. The average final price is \$132.55. The average auction attracts 17 bids, including rebids of users who have been outbid, and the average number of auction participants is 8.4. Items are always brand new in the BIN listings; for the auctions, 10.8% of the listing titles indicate prior use with the words "mint," "used," or "like new." About 28% of the titles imply that standard bonus tapes or videos are included; the professional retailers always include both extras. Thus, the goods auctioned on eBay are, if anything, of somewhat lower quality than those sold on eBay by the fixed-price retailers, rendering the overbidding proxy even more conservative.

[15] The exact time period is 2/11/2004 to 9/6/2004. Data is missing on the days from 7/16/2004 to 7/24/2004 since eBay changed the data format. Two auctions during which a professional listing was not always available (between 23:15 p.m. PDT on 8/14/2004 to 8:48 p.m. on 8/20/2004) were also dropped.

[16] The identification strategy refines the approach of Ariely and Simonson (2003), who compare auction and retail prices on other online sites, and Ashenfelter and Genesove (1992) who compare prices in real-estate auctions and in face-to-face negotiations.

[17] Later, the price was raised to \$139.95. The game has since been discontinued.

Figure 4: Share of Overbid Auctions for Different Goods



Note: This figure uses the cross-sectional eBay data of Malmendier and Lee (2011). It contains a cross-section of $1,929$ different auctions downloaded at the same point in time (on June 6, 2008) for various categories. The figure depicts the proportion of overbid auctions by item category. The leftmost column shows the percent of auction prices above the BIN in the Cashflow 101 data. The other columns show the percent of auction prices above the corresponding BIN in the cross-sectional data, split by item category.

The main result reported by Malmendier and Lee (2011) is that 43% of auctions end up at prices above the simultaneously available fixed price. This result is *not* due to differences in shipping costs: if those differences are also accounted for, 73% of auctions end above the fixed price. Simple regressions show that overbidding is not explained by differences in item quality or seller reputation. The amount of overbidding is significant: 27% of auctions exceed the fixed price by more than \$10, and 16% by more than \$20. Even the average auction price exceeds the fixed price, which helps rule out rational explanations based on switching costs, since a rational buyer only enters the auction if the expected price is below the fixed price.

Do these results extend to other goods besides board games? Figure 4 addresses this question using a second, broader data set, also detailed in Malmendier and Lee (2011), that contains a cross-section of $1,929$ different auctions, which downloaded at the same point in time (on June 6, 2008) and range from electronics to sports equipment. The figure shows the proportion of overbid auctions by item category, with Cashflow 101 depicted as the baseline on the left. Cashflow 101 is not an outlier: the figure shows that overbidding in auctions is common across a wide variety of goods, with the proportion of overbid auctions ranging between 30% and 60% for most categories.

In summary, the main findings of Malmendier and Lee (2011) are another piece of evidence on overbidding in auctions, in this case explicitly in comparison to a fixed-price market.

5.2 Amplification in Revenue and Allocations

Given the presence of overbidding in this market, we can explore the specific predictions of our model about amplification. A key prediction, formally established in Propositions 1 and 3, is that a few biased agents can have amplified effects on revenue, because they end up setting the price with disproportionately high frequency.

Table 2 presents evidence on amplification using the detailed bidder- and bid-level data of the board game Cashflow 101, available for 138 auctions that have on average 8.4 participants. (See the bidder- and bid-level summary statistics in Panels B and C of Table 1.) The fraction of buyers who overbid in these auctions is only about 17%; however, 43% of auctions are overbid in the sense that the final price is above the fixed price at which the good is available. The effect of behavioral agents is thus amplified by a factor of $0.43/0.17 = 2.5$ relative to their share in the population. This effect is a manifestation of the fishing mechanism: high-bidding behavioral agents are more likely to set the price in these auctions. The findings in Table 2 thus provide strong evidence for the basic amplification prediction.[18]

How well does our model explain the magnitude of amplification observed in the data? To answer, we calibrate the model by setting the share of behavioral buyers in the population equal

[18] The fraction of bids that exceed the fixed price (11%) is lower than the fraction of buyers who overbid (17%). The difference is due to the fact that in the data, unlike in the model, some buyers bid multiple times. The right comparison with the theory is to use the fraction of buyers, treating their final bid as equivalent to the single bid agents can submit in the model.

to $\pi^* = 17\%$ and then calculate the probability of at least two fools for each auction in our sample. The share of auctions with at least two fools is predicted to be $q_2^n(\pi^*) = 41\%$.[19] This is very close to the 43% of overbid auctions we observe in the data. In fact, we cannot reject the hypothesis that the sample proportion of 43% is the same as the predicted proportion ($p = 0.59$). The reason for this close quantitative fit is straightforward. Both in the theory and in the field, we count the share of auctions with at least two fools, given the baseline share of fools in the population. If fools are distributed evenly across auctions in the data, the law of large numbers predicts that the share of overbid auctions should be approximately $q_2^n(\pi^*)$. The fact that this prediction is verified in the data provides evidence for one of our main assumptions and thus external validity for our model.

We next turn to amplification in allocations. As highlighted by Corollary 4, our model predicts that biased agents win with an amplified probability relative to their population share. In the model the magnitude of this amplification is governed by $q_1^n(\pi^*)$. In the data the bias of the agent winning the good is not observed, but we can calibrate the formula using the above parameters to obtain $q_1^n(\pi^*) \approx 73\%$. The fact that our calibrated model closely matched the probability of at least two biased bidders, and the implication that biased bidders appear to be evenly distributed, suggests that this prediction is also likely to be precise. Thus even though only 17% of agents are biased, they win 73% of auctions. This finding indicates that there is significant potential for misallocation due to the presence of biased bidders. We conclude that the above results strongly support the basic amplification mechanism identified by the model.

5.3 Auctions versus Fixed-Price Markets

We next consider the prediction in Propositions 2 and 4 that, in the presence of biased bidders, sellers prefer auctions over other selling mechanisms. In both propositions, the preference for auctions requires a strictly positive, albeit not high share π of interested buyers who are subject to overvaluation bias.

Empirical evidence for this prediction is harder to generate as it requires estimates of (1) the share of behavioral buyers (fools) for different goods, and (2) the market share of auctions and fixed

[19] If we use the average auction size of $n = 8.4$, the share of auctions with at least two fools is predicted to be $q_2^n(\pi^*) = 43\%$. This is almost exactly what we observe in the data.

prices for these goods. We discuss some suggestive evidence, including new facts, that is consistent with the prediction and provides a starting point for future empirical investigations.

Specifically, we focus on evidence that contrasts goods for which it seems implausible that biases would induce buyers to overbid and overpay ($\pi = 0$), with other goods, for which overvaluation appears more plausible ($\pi > 0$). Candidates for the former goods are commodity-type items that are new, have a model number or other clear identification, and salient market prices. For these items, it might be less plausible for a buyer to overbid or fail to note the standard (outside) price. Candidates for the latter type of items are collectibles, memorabilia and other used, unique, or antiques items. These items provide a greater scope for differences and mistakes in valuation.

We explore how the frequencies of auction- versus fixed-price based selling mechanisms compare across markets for either type of goods.

First, we return to the broad, cross-sectional eBay data from Malmendier and Lee (2011), which we discussed above and displayed in Figure 4. For the discussion here, we calculate the absolute and relative frequencies of both auction and fixed-price listings for all items in the data, split into 33 finer categories. Table 3 displays the results, sorted by decreasing auction frequency. The cross-sectional draw reveals that commodity-type items, such as tickets, cameras, cell phones, PDAs, consumer electronics, books, and computers, have the lowest relative auction frequency. On the other end of the spectrum, items such as stamps, pottery, antiques, art, coins and paper money, collectibles, and sports memorabilia have the highest relative auction frequency. The much higher relative frequency of auctions in the latter groups is consistent with the model we propose.

A similar pattern characterizes online auctions more broadly: Lucking-Reiley (2000) provides evidence of the type of items sold and the revenues generated on 142 Internet auction sites. He finds that by far the most common type of item is collectibles, which includes antiques, celebrity memorabilia, stamps, coins, toys, and trading cards. Lucking-Reiley (2000) also finds that most goods auctioned online are used items that are being resold. Relatedly, evidence from Einav, Farronato, Levin, and Sundaresan (2018) confirms the impression that idiosyncratic, expensive items are commonly sold via auction, more so than standard items.[20]

[20] Whereas this paper models auctions as a trade-off between higher cost for auctions with a potentially higher reward, Einav, Farronato, Levin, and Sundaresan (2018) models auctions as a trade-off between a higher sale proba-

Finally, the same picture emerges from data of traditional, offline auctions, which tend to sell collectibles and similar items rather than commodities. We survey the categories of objects sold by five of the largest auction houses: Christie's, Sotheby's, Bonhams 1793, Stockholm Auction House, and Lyon and Turnbull. As shown in the comprehensive listing of Appendix-Table B.1, items such as art and antiquities, furniture, collectibles, and photographs are indeed among the most common categories.

Our model implies not only that profit-maximizing sellers should prefer auctions if they can expect to attract a non-zero fraction of biased buyers ($\pi > 0$), but also that a significant fraction of items should be sold at inflated prices. Sotheby's website also provides some suggestive evidence, based on the ex-ante appraisals of auction objects as well as their final prices. Bids above the appraisal are, of course, not evidence of overbidding; but they indicate that items sold in auction houses are perceptible to a wide range of valuations, including potential overestimates.

We collect the online descriptions, selling prices, and appraisal ranges (high and low valuations) of all auctions conducted between January and July 2007.[21] After discarding four auctions that lack catalog descriptions and online appraisals, we obtain a sample of 43,107 items sold in 178 auctions. We classify items as antiques, art, books, jewelry, sports, stamps, and wine. We compute the frequencies with which items are sold at a price above the highest appraisal, as well as the price/appraisal ratios for both the highest and the lowest appraisal. Table 4 summarizes these data and calculations.

We find that, of the 178 auctions, 168 generated revenues above the most optimistic expectation benchmark, i. e., the highest appraisal. 14.71% of all items sell for double the amount of the most optimistic appraisal, with the maximum being a 100-fold multiple. In other words, the final bid overwhelmingly exceeds the most optimistic assessment in the vast majority of traditional auctions of collectibles and similar items, consistent with the notion of possible overbidding as well as targetted use of auctions in such circumstances and for such items.

These findings are suggestive of our predictions in the sense that used, unique, or antique items

bility for auctions with a lower sale price. They document a decline in auctions and find that it is mainly driven by an increasing relative preference for convenience shopping, rather than changes in the composition of buyers, sellers, or items.

[21] Revenues in foreign currencies are converted to USD using the conversion rates on June 20, 2008.

such as art, coins, and other collectibles and memorabilia seem plausible candidates for attracting behavioral bidders, and the use of auctions is consistent with the idea that sellers choose auctions in part to take advantage of buyer biases. Needless to say that there are alternative explanations such as higher asymmetric information or greater heterogeneity in values. While the evidence on the choice of auctions versus markets documented here is suggestive, it complements our findings on overbidding and amplification and is supportive of the view that sellers choose the auction format where the probability of behavioral mistakes is higher.

5.4 Attracting Additional Bidders

Corollaries 3 and 7 suggest that, in the presence of biased agents, attracting additional buyers is especially profitable for sellers in auction-like markets. Here we discuss some anecdotal evidence consistent with this prediction.

One effective strategy for attracting buyers is setting a low initial price, as evidenced by both experimental (Ku, Galinsky, and Murnighan 2006) and eBay field studies (Simonsohn and Ariely 2008). This strategy appears popular also in auction-like markets. For example, consider the US housing market in which buyers can be pitted against one another. Bucchianeri and Minson (2013) analyze the optimal pricing strategy in this market by surveying realtors and examining web evidence. They find that realtors frequently suggest underpricing to start a bidding war.[22] Similarly, in the home auction market of New South Wales, Australia, underquoting was extraordinarily successful at attracting additional buyers and creating bidding fever. In fact, underquoting became so problematic that the Australian government had to pass a law prohibiting it.[23] A related strategy for sellers, common in housing markets in the United States (Chava and Yao 2017) as well as Sweden (Repetto and Solis 2017), is to take advantage of left digit bias to increase interest. Consistent with our model, all of these findings point to a correlation between the presence of behavioral biases and strategies to attract additional buyers in auctions.

[22] They also suggest that this may be due to an agency issues rather than agents believing it is an optimal strategy for maximizing home revenue.

[23] See fairtrading.nsw.gov.au/ftw/About_us/Legislation/Changes_to_legislation/Underquoting_reforms; abc.net.au/news/2017-04-29/real-estate-underquoting-still-prevalent-despite-crackdown/8481846; propertyupdate.com.au/agents-underquote-property-prices-beat-game-2; thenewdaily.com.au/money/property/2017/05/12/underquoting.

6 Conclusion

This paper develops a simple model to study the effect of psychological biases inducing overvaluation in both auctions and fixed-price markets. We find that both market mechanisms can amplify rather than ameliorate the effect of biases on prices, allocation, and ultimately welfare. The amplification effect is stronger in auctions in the sense that auctions take advantage of even a few agents with behavioral biases to increase revenues. Further amplification arises when sellers can choose their preferred selling mechanism, as they switch from efficient fixed-price markets to inefficient auctions. Sellers may also make inefficient production and marketing decisions as a result. Evidence from various markets is consistent with these predictions.

Our findings question the received wisdom of neoclassical economics that markets attenuate the effect of behavioral biases. Our results may explain part of the popularity of auctions in allocating goods in practice, particularly when their market value is hard to assess.

We conclude with two questions to be explored in future research. The first is closely related to the auction and market environments studied in this paper. How large are the welfare costs of overbidding in practice? Our model suggests that they can be significant. However, to settle this question, careful measurement of bidder behavior across different environments is needed. Our second question is broader. What other mechanisms amplify the impact of behavioral biases? We know from the behavioral industrial organization literature that sellers often cater to biases in contract design. Do these or other mechanisms result in amplification, analogous to our results? Early hints come from the behavioral bilateral trading literature (Saran 2011, Crawford 2019); answering these questions in broader contexts can help make progress with the larger agenda of understanding behavioral agents in markets.

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Table 1: Summary Statistics, Cash-Flow 101 Data
Panel A: Auction-Level Data

Variable	Obs.	Mean	Std. Dev.	Min.	Max.
Starting Price	165	46.14	43.81	0.01	150
Final Price	166	132.55	17.03	81.00	179.30
Shipping Cost	139	12.51	3.75	4.95	20.00
Total Price	139	144.68	15.29	110.99	185.50
Number of Bids	166	16.91	9.13	1	39
Number of Bidders	139	8.36	3.87	1	18
Feedback Score Buyer	166	36.84	102.99	0	990
Feedback Score Seller	166	261.95	1,432.95	0	14,730
Positive Feedback Percentage Seller	166	62.92	48.11	0	100
Auction Length [in days]	166	6.30	1.72	1	10
one day	166	1.20%			
three days	166	11.45%			
five days	166	16.87%			
seven days	166	65.06%			
ten days	166	5.42%			
Auction Ending Weekday					
Monday	166	11.45%			
Tuesday	166	7.83%			
Wednesday	166	15.66%			
Thursday	166	12.05%			
Friday	166	9.64%			
Saturday	166	18.67%			
Sunday	166	24.70%			
Auction Starting Hour	166	14.78	5.20	0	23
Auction Ending Hour	166	14.80	5.21	0	23
Prime Time	166	34.34%			
Title New	166	28.31%			
Title Used	166	10.84%			
Title Bonus Tapes/Video	166	21.08%			
Explicit195	166	30.72%			

Note: The sample period is 02/11/2004 to 09/06/2004. Final Price is the price paid by the winner excluding shipping costs; it is equal to the second-highest bid plus the bid increment. Shipping Cost is the flat-rate shipping cost set by the seller. Total Price is the sum of Final Price and Shipping Cost. Auction Starting and Ending Hours are defined as 0 for the time interval from 12 am to 1 am, 1 for the time interval from 1 am to 2 am etc. Prime Time is a dummy variable and equal to 1 if the auction ends between 3 pm and 7 pm PDT. Delivery Insurance is a dummy variable and equal to 1 if any delivery insurance is available. Title New is a dummy and equal to 1 if the title indicates that the item is new. Title Used is a dummy and equal to 1 if the title indicates that the item is used. Title Bonus Tapes/Video is a dummy and equal to 1 if the title indicates that the bonus tapes or videos are included. Explicit195 is a dummy variable equal to 1 if the item description mentions the $195 manufacturer price.

Table 1 (*continued*)

Panel B: Bidder-Level Data

Variable	Obs.	Mean	Std. Dev.	Min.	Max.
Number of auctions per bidder	807	1.44	1.25	1	17
Number of bids per bidder (total)	807	2.92	3.35	1	33
Number of bids per bidder (per auction)	807	2.13	1.85	1	22
Average bid per bidder [in $]	807	87.96	38.34	0.01	175.0
Maximum bid per bidder [in $]	807	95.14	39.33	0.01	177.5
Winning frequency per bidder (total)	807	0.17	0.38	0	2
Winning frequency per bidder (per auction)	807	0.15	0.34	0	1

Note: Bids are submitted bids, except in the case of the winning bid which is displayed as the winning price (the second-highest bid plus the appropriate increment).

Panel C: Bid-Level Data

Variable	Obs.	Mean	Std. Dev.	Min.	Max.
Bid value [in $]	2,353	87.94	36.61	0.01	177.5
Bid price outstanding [in $]	2,353	83.99	38.07	0.01	177.5
Leading bid [in $]	2,353	93.76	35.18	0.01	177.5
Feedback Score Buyer	2,353	32.40	104.65	-1	1,378
Feedback Score Seller	2,353	273.23	1422.55	0	14,730
Positive Feedback Percentage Seller	2,353	64.72	47.40	0	100
Starting time of auction	2,353	15.63	4.91	0.28	23.06
Ending time of auction	2,353	15.68	4.93	0.28	23.41
Bidding time	2,353	13.70	5.54	0.20	24.00
Last-minute bids					
during the last 60 minutes	2,353	6.25%			
during the last 10 minutes	2,353	4.25%			
during the last 5 minutes	2,353	3.48%			
Bid on auction with Explicit195	2,353	0.32	0.47	0	1
Bid on auction with delivery insurance option	2,353	0.46	0.50	0	1
Bids on auction with bonus tapes/videos	2,353	0.25	0.43	0	1

Table 2: Amplified Effect of Overbidders

		Observations	(Percent)
Auction-level sample			
Does the auction end up overbid?	No	78	56.52%
	Yes	60	43.48%
Total		138	100.00%
Bidder-level sample			
Does the bidder ever overbid?	No	670	83.02%
	Yes	137	16.98%
Total		807	100.00%
Bid-level sample			
Is the bid an over-bid?	No	2,101	89.29%
	Yes	252	10.71%
Total		2,353	100.00%

Note: Data is hand-collected from eBay for the Cashflow 101 game. Data was collected between 2/11/2004 to 9/6/2004. Data is missing on the days from 7/16/2004 to 7/24/2004 since eBay changed the data format. Two auctions during which a professional listing was not always available (between 23:15 p.m. PDT on 8/14/2004 to 8:48 p.m. on 8/20/2004) were also dropped. Overbidding is tabulated by observation at the auction, bidder, and bid levels. Overbidding is defined relative to the buy-it-now price (without shipping costs).

Table 3: Frequencies of Auctions versus Fixed Price

Category	Auction items		Fixed-price items	
	Number	Percent	Number	Percent
Stamps	151,720	93.3%	10,822	6.7%
Pottery & Glass	190,566	88.7%	24,397	11.3%
Antiques	169,983	86.5%	26,468	13.5%
Art	148,665	78.6%	40,411	21.4%
Coins & Paper Money	203,061	84.4%	37,608	15.6%
Collectibles	1,293,036	79.6%	330,434	20.4%
Sports Mem, Cards & Fan Shop	571,730	77.7%	164,440	22.3%
Clothing, Shoes & Accessories	2,198,045	68.9%	992,488	31.1%
Dolls & Bears	111,512	76.6%	34,087	23.4%
Music	390,900	74.5%	133,600	25.5%
Jewelry & Watches	980,948	74.8%	330,332	25.2%
Gift Certificates	9,874	75.6%	3,188	24.4%
Entertainment Memorabilia	157,926	68.6%	72,396	31.4%
Toys & Hobbies	568,978	72.8%	213,049	27.2%
Crafts	268,859	72.9%	100,013	27.1%
Sporting Goods	356,493	63.9%	201,339	36.1%
Video Games	216,644	56.6%	165,938	43.4%
Everything Else	111,625	69.6%	48,668	30.4%
Baby	45,658	64.4%	25,190	35.6%
Business & Industrial	195,441	59.6%	132,232	40.4%
Home & Garden	532,393	57.8%	388,437	42.2%
DVDs & Movies	315,195	54.7%	260,995	45.3%
Travel	7,727	56.9%	5,854	43.1%
Musical Instruments	104,559	54.9%	85,984	45.1%
Real Estate	2,939	75.7%	943	24.3%
Tickets	32,352	50.8%	31,331	49.2%
Health & Beauty	254,537	54.4%	212,988	45.6%
Cameras & Photo	134,871	44.2%	170,381	55.8%
Cell Phones & PDAs	237,793	37.4%	398,147	62.6%
Consumer Electronics	199,825	42.8%	266,653	57.2%
Books	495,422	47.0%	557,866	53.0%
Computers & Networking	251,442	40.2%	374,227	59.8%
Specialty Services	2,656	24.3%	8,260	75.7%

Note: Absolute and relative frequencies of auction listings and fixed-price listings on eBay by category as of June 6, 2008 (22:17pm PDT). Number denotes the absolute number of listings; Percent the percentage of lisitings of a given type out of all auction and fixed-price listings in the respective category. Categories are sorted by decreasing percentages of auction listings. Source: `http://listings.ebay.com`.

Table 4: Bidding Relative to Appraisal

	Number (Percent)		Revenues in USD (in %)		Price above high valuation	Ratio of price to high appraisal	Ratio of price to low appraisal
Antiques	23	(12.92%)	$90,222,465	(3.16%)	52.33%	1.43	2.12
Art	117	(65.73%)	$2,535,543,690	(88.82%)	57.25%	1.45	2.09
Books	8	(4.49%	$43,276,989	(0.94%)	53.55%	1.52	2.17
Jewelry	18	(10.11%)	$167,042,481	(5.85%)	56.47%	1.30	1.82
Sports	2	(1.12%)	$14,151,555	(0.50%)	48.82%	1.14	1.74
Stamps	2	(1.12%)	$3,136,188	(0.11%)	32.51%	1.10	1.40
Wine	8	(4.49%)	$17,846,897	(0.63%)	65.53%	1.35	1.81

Note: The table presents information based on online descriptions, selling prices, and appraisal ranges (high and low valuations) of all auctions from Sotheby's conducted between January and July 2007. We discard four auctions that lack catalog descriptions and online appraisals. Categories of objects sold by five of the largest auction houses, are used. Items classified as antiques, art, books, jewelry, sports, stamps, and wine. Revenues in foreign currencies are converted to USD using the conversion rates on June 20, 2008. Price above high valuation is by item. Ratios presented are the average across auctions.

Appendix A: Proofs

We first establish two lemmas, which bound the probabilities of having at least two and at least one biased agent out of n buyers, $q_2^n(\pi)$ and $q_1^n(\pi)$. Using these lemmas we then present proofs of all propositions and corollaries in the order in which they appear in the main text. Here and below we denote the base of the natural logarithm by e.

Lemma 1. *If $\pi(n-1) \geq a$ for some $0 < a \leq 1$, then $q_2^n(\pi) > a^2 \cdot \frac{e-2}{e}$.*

Proof. Consider $q_2^n(\pi) = 1-(1-\pi)^n - n\pi(1-\pi)^{n-1}$ as a function of n defined not only for integers but for all $n > 0$. This function is strictly increasing in n for $n > 0$. This fact can be directly verified by differentiation and some calculations, and is also intuitive as the probability of at least two biased agents should be higher when the total number of agents n is larger. The assumption $\pi(n-1) \geq a$, or $n \geq 1 + a/\pi$, implies that $q_2^n(\pi) \geq q_2^{1+a/\pi}(\pi)$. Moreover,

$$q_2^{1+a/\pi}(\pi) = 1 - (1-\pi)^{1+a/\pi} - (\pi + a)(1-\pi)^{a/\pi} = 1 - (1-\pi)^{a/\pi}(1+a). \tag{1}$$

This is thus a lower bound for $q_2^n(\pi)$.

To develop a bound that does not depend on π, we next show that $1-(1-t)^{a/t}(1+a)$, which is our bound when $t = \pi$, is an increasing function of t in the range $t > 0$. This will imply that we can further bound (1) from below with the limit of $1-(1-t)^{a/t}(1+a)$ as t goes to zero. Note that

$$\begin{aligned}\frac{d((1-t)^{a/t})}{dt} &= (1-t)^{a/t}[-\frac{a}{t^2}\log(1-t) - \frac{a}{(1-t)t}] \\ &= (1-t)^{a/t}(-\frac{a}{t})[\frac{1}{t}\log(1-t) + \frac{1}{1-t}],\end{aligned}$$

and the Taylor expansion of the term in the bracket around $t = 0$ is

$$\frac{1}{t}\log(1-t) + \frac{1}{1-t} = \frac{1}{2}t + \frac{2}{3}t^2 + \frac{3}{4}t^3 + ... + \frac{k}{k+1}t^k + ... > 0,$$

so that

$$\frac{d((1-t)^{a/t})}{dt} < 0,$$

as claimed. Thus $q_2^n(\pi)$ is strictly bounded from below by

$$\lim_{t\to 0} 1 - (1-t)^{a/t}(1+a) = 1 - e^{-a}(1+a). \tag{2}$$

Finally we develop a quadratic lower bound for this expression. Consider $h(z) = 1 - e^{-z}(1+z) - \dfrac{z^2(e-2)}{e}$. Inspecting the derivative of $h(.)$ it is easy to verify that there is some $z^* \in (0,1)$ such that for $z \in (0, z^*)$, $h'(z) > 0$ and for $z \in (z^*, 1)$, $h'(z) < 0$. Since $h(0) = 0$ and $h(1) = 0$, we have $h(z) > 0$ for all $z \in (0,1)$. Using this for $z = a$ implies that $1 - e^{-a}(1+a) > a^2 \cdot (e-2)/e$ and the claim follows. □

Lemma 2. *If $0 < \pi \leq 1/n$, then $n\pi > q_1^n(\pi) > n\pi\frac{e-1}{e}$.*

Proof. Recall that $q_1^n(\pi) = 1-(1-\pi)^n$. For $\pi \in [0,1]$ the function $(1-\pi)^n$ is convex and decreasing. Its derivative at $\pi = 0$ is $-n$. It follows that $(1-\pi)^n \geq 1 - n\pi$, establishing the upper bound. For the lower bound, note that for $\pi \in (0, 1/n]$

$$(1-\pi)^n \leq (1-n\pi)\cdot 1 + (n\pi)\cdot(1-1/n)^n < 1 - n\pi + (n\pi)1/e = 1 - \frac{e-1}{e}n\pi, \tag{3}$$

where the first step uses Jensen's inequality for the convex $(1-\pi)^n$ and the random variable that takes on value 0 with probability $1-n\pi$ and value $1/n$ with probability $n\pi$ and thus has expected value π; and the second step uses the observation established in the proof of Lemma 1 that $(1-1/n)^n$ is increasing in n and converges to $1/e$. □

Proof of Proposition 1. (a) Let $\kappa_1 = e/(e-2)$.

Case (i): $\pi(n-1) \leq 1$. Then for $a = \pi(n-1)$, the assumption of Lemma 1 is satisfied, and the Lemma implies that the revenue impact of fools is at least $(n-1)^2\pi^2 \cdot \frac{e-2}{e}(v_B - v)$. This in term is bounded from below by $\pi(v_B - v)$ when $1/(n-1)^2 \cdot e/(e-2) \leq \pi$, which is the range of π considered in Proposition 1(a).

Case (ii): $\pi(n-1) > 1$. For $n = 3, 4, 5$ one can directly check that $q_2^n(\pi) > \pi$ holds if $1/(n-1) \leq \pi < 1$. Next note that for a given π, if $q_2^n(\pi) \geq \pi$ for some n, then the same holds when n is replaced by any $n' > n$ since $q_2^{n'}(\pi) - q_2^n(\pi) \geq 0$. Combining this observation with the results for $n = 3, 4, 5$ it follows that when $\pi \geq 1/4$, the desired inequality holds for all n satisfying $\pi(n-1) \geq 1$. Finally, using Lemma 1 with $a = 1$ implies the revenue gain is at least $\frac{e-2}{e}(v_B - v)$ so the result also holds if $\pi < (e-2)/e$, and because $(e-2)/e > 1/4$ the proof is complete.

(b) Let $\kappa_2 = \log(v_B/(v_B - v))$. Amplification follows if $(1-(1-\pi)^n)v_B - v > \pi(v_B - v)$ or equivalently $(1-\pi)(v_B - v) > (1-\pi)^n v_B$, which holds if $\log((v_B - v)/v_B) > (n-1)\log(1-\pi)$. Because $-\pi > \log(1-\pi)$, the condition is satisfied if $\log((v_B - v)/v_B) > -(n-1)\pi$ or equivalently if $\pi > \kappa_2/(n-1)$ as stated in the Proposition.

For completeness we also show that for $\pi \leq (1/n)(v/v_B)$ the fixed price market does not generate amplification. Indeed, in this case by Lemma 2 we have $(1-(1-\pi)^n)v_B < n\pi v_B \leq v$ and hence the seller sets the price at v.

Proof of Proposition 2. Let $\kappa_3 = v_B/v$. In the auction, for $0 < \pi$ biased agents increase revenue by some positive amount. In the fixed price market, by the last paragraph in the proof of Proposition 1(b), when $\pi \leq 1/(\kappa_3 n)$ the seller sets the price at v. Thus biased agents do not increase revenue at all.

Proof of Corollary 1. First note that when $\pi = 0$, the profit is $v - c_a$ in the auction and $v - c_f$ in the fixed price market. Hence for $\pi = 0$, when $c_a - c_f > 0$ the seller prefers the fixed-price market. Now suppose that $\pi = (1/n)v/v_B$. Then by the proof of Proposition 1(b), in the fixed-price market the seller still sets the price to be v and earns profit $v - c_f$. But in the auction, the profit is at least $v + q_2^n(\pi)(v_B - v) - c_a \geq v + \frac{e-2}{e}\frac{(n-1)^2}{n^2}\frac{v^2}{v_B^2}(v - v_B) - c_a$ by Lemma 1. Using $n \geq 2$, the profit of the auction is higher if $c_a - c_f < \frac{e-2}{4e}\frac{v^2}{v_B^2}(v - v_B)$. Since payoff functions are continuous, the fact that the result holds for this value of π implies that it also holds in some neighborhood, i.e., for a range of π values.

Proof of Corollary 2. When $\pi = 0$ it is immediate that the seller chooses the fixed-price market and produces efficiently. Now suppose $\pi = (1/n)v/v_B$. In the fixed-price mechanism the seller sets the price at v and invests if $k_p \leq v - c_f$. But if the seller can choose the mechanism, then by the proof of Corollary 1 she chooses the auction, earns higher profits, and hence sets a cutoff for production that exceeds the efficient level. By continuity, the result holds for a range of π.

Proof of Corollary 3. Suppose $\pi = [1/(n+1)](v/v_B)$. In the fixed-price market with n buyers, by the proof of Corollary 1 the benefit of recruiting one more buyer is zero, so the seller does not recruit. If the seller can choose the market mechanism, then it is easy to see, using steps analogous to those in the proof of Corollary 1, that when $c_a - c_f$ is small enough, the seller prefers the auction. In that auction, the benefit of recruiting one more buyer is

$$[q_2^{n+1}(\pi) - q_2^n(\pi)](v_B - v) = (1-\pi)^{n-1}\pi^2 n(v_B - v).$$

If $k_b < (1-\pi)^{n-1}\pi^2 n(v_B - v)$ for this value of π, the seller will want to recruit. And by continuity the result also holds for a range of π in the neighborhood of this value.

Proof of Proposition 3. For the claim on amplification in the auction, set a to satisfy

$$\frac{a^2(e-2)}{e} = \frac{\pi(v_B - v_L)}{v_B - v_H}.$$

If this choice meets the assumptions of Lemma 1, then Lemma 1 implies that the change in expected revenue from increasing the share of fools from 0 to π is strictly larger than $\pi(v_B - v_L)$. The conditions of the Lemma are that $a \leq 1$ and $\pi(n-1) \geq a$. Setting $\tilde{\kappa}_1 = \frac{e}{e-2} \cdot \frac{v_B - v_L}{v_B - v_H}$, the former is equivalent to $\pi \leq 1/(\tilde{\kappa}_1)$, and the latter is equivalent to

$$\pi^2(n-1)^2 \geq \frac{e}{e-2} \cdot \frac{\pi(v_B - v_L)}{v_B - v_H},$$

or equivalently $\pi \geq \tilde{\kappa}_1/(n-1)^2$. These are the inequalities in the statement of the Proposition.

Next consider the fixed-price market. Amplification holds if $q_1^n(\pi)v_B - v_H > \pi(v_B - v_L)$ which, after writing out $q_1^n(\pi)$, is equivalent to

$$v_B - v_H > (1-\pi)^n v_B + \pi(v_B - v_L).$$

Now assume that $\pi \leq (1/2)(v_B - v_H)/(v_B - v_L)$. Then we can bound the second term on the right-hand side with 1/2 times the left hand side, so the condition would hold if $(1/2)(v_B - v_H) > (1-\pi)^n v_B$. Taking logs, we need π to satisfy

$$\log((v_B - v_H)/(2v_B)) > n \log(1-\pi).$$

Because $-\pi > \log(1-\pi)$, this condition is in turn satisfied if $\log((v_B - v_H)/(2v_B)) \geq -n\pi$, that is, if we also assume

$$\log(2v_B/(v_B - v_H))/n \leq \pi.$$

Now set $\tilde{\kappa}_2 = \max(\log(2v_B/(v_B - v_H)), 2(v_B - v_L)/(v_B - v_H))$. Then for $\tilde{\kappa}_2/(n-1) \leq \pi \leq 1/\tilde{\kappa}_2$ both assumptions imposed on π in the above derivation hold, and thus the fixed-price market generates amplification.

Proof of Corollary 4. Start with the auction. Because the revenue gain from a biased agent setting the price is at most $v_B - v_L$, Proposition 3 implies that the price is v_B with probability higher than π; that is, $q_2^n(\pi) > \pi$. Since $q_1^n(\pi) \geq q_2^n(\pi)$ the statement follows.

Now consider the fixed-price market. The proof of Proposition 3 showed that in the assumed range for π, we have $q_1^n(\pi)v_B - v_H > \pi(v_B - v_L)$. This implies $q_1^n(\pi)v_B - q_1^n(\pi)v_L > \pi(v_B - v_L)$, which implies $q_1^n(\pi) > \pi$, so the probability of a biased buyer exceeds π. And because the proof of Proposition 3 also showed that, in the given range, the seller sets the price at v_B, only biased buyers can get the good.

Proof of Proposition 4. Begin with the fixed-price market. Let v_M be the median of the distribution F, and suppose $\pi \leq \frac{v_M}{2v_B - v_M} \cdot \frac{1}{n}$. We prove that then the seller does not set the price at v_B. We know $v_M > v_L$ because f is bounded away from zero. A strategy always available to the seller is to set $p = v_M$ and sell with probability $1 - (1-\pi)^n \frac{1}{2^n}$. Thus setting v_B is not optimal if

$$v_M(1 - (1-\pi)^n \frac{1}{2^n}) \geq v_B(1 - (1-\pi)^n),$$

or equivalently

$$(v_B - \frac{v_M}{2^n})(1-\pi)^n \geq v_B - v_M.$$

The left-hand side is strictly bounded from below by $(v_B - v_M/2)(1 - n\pi)$, and it follows that when $\pi \leq \frac{v_M}{2v_B - v_M}\frac{1}{n}$, v_B is not the optimal price. It is also immediate that setting a price $p \in [v_H, v_B)$ is strictly dominated by setting the price at v_B, and hence is also not optimal when π is in the above range. And setting the price at $p \in (0, v_L)$ is strictly dominated by setting the price at v_L. Thus when $\pi \leq \frac{v_M}{2v_B - v_M} \cdot \frac{1}{n}$ the optimal price is either v_L or interior in $[v_L, v_H]$.

Now consider the effect of a marginal increase in π on revenue. When the optimal price satisfies the first-order condition of (1), this effect can be computed directly from (1) using the envelope theorem as

$$n(1-\pi)^{n-1} pF(p)^n. \tag{4}$$

When the optimal price does not satisfy the first-order condition, from the above argument that price must be v_L, and because the marginal cost of increasing the price is strictly larger than its marginal benefit, it must also be true that a small increase in π does not change the optimal price. We conclude that (4) is an upper bound for the marginal effect of π as long as $\pi \leq \frac{v_M}{2v_B - v_M} \cdot \frac{1}{n}$.

Our next step is to derive an upper bound for $F(p)^n$ which is the key term in (4). When $p = v_L$, zero is an upper bound. When $p > v_L$, we use the first-order condition, which is

$$1 - (1-\pi)^n F(p)^n = (1-\pi)^n npf(p)F(p)^{n-1}. \tag{5}$$

This condition implies that for the optimal p

$$F(p)^n \leq \frac{1}{(1+npf(p))(1-\pi)^n}. \tag{6}$$

Combining (4) and (6), the marginal revenue effect of increasing π is at most

$$\frac{n(1-\pi)^{n-1}p}{(1+npf(p))(1-\pi)^n} < \frac{n(1-\pi)^{n-1}p}{(npf(p))(1-\pi)^n} \leq \frac{1}{\underline{f}(1-\pi)} < \frac{1}{2\underline{f}},$$

where we used that $\pi < 1/2$, which holds because $\pi \leq \frac{v_M}{2v_B - v_M} \cdot \frac{1}{n}$ and $n \geq 2$. It follows that the overall revenue effect of increasing the share of biased agents from zero to some value π, which is the integral of the marginal effects, is less than $\pi/(2\underline{f})$.

Now consider the auction. Using Lemma 1 with $a = (n-1)\pi$ (where $a < 1$ because $\pi \leq \frac{v_M}{2v_B - v_M} \cdot \frac{1}{n}$), the revenue effect of biased agents in the auction is at least

$$\frac{e-2}{e}(n-1)^2\pi^2(v_B - v_H).$$

Comparing this to the upper bound for the fixed price market, biased agents generate more revenue in the auction than in the market if

$$\pi \geq \frac{e}{e-2} \cdot \frac{1}{2\underline{f}(v_B - v_H)} \cdot \frac{1}{(n-1)^2}.$$

Thus setting $\tilde{\kappa}_3 = \max(\frac{e}{e-2}\frac{1}{2\underline{f}(v_B - v_H)}, \frac{2v_B - v_M}{v_M})$ implies that for $\tilde{\kappa}_3/(n-1)^2 \leq \pi \leq 1/(\tilde{\kappa}_3 n)$, biased agents generate more revenue in the auction than in the market.

Proof of Corollary 5 Using $\tilde{\kappa}_3$ which was defined in the proof of proposition 4, let n be large enough that $\frac{\tilde{\kappa}_3}{(n-1)^2} < \frac{1}{\tilde{\kappa}_3 n}$. Denoting revenue in the auction and in the fixed-price market by $R_a(\pi)$ and $R_f(\pi)$, let $c_1(n) = R_a(0) - R_f(0)$ and

$$c_2(n) = R_a(\frac{1}{\tilde{\kappa}_3 n}) - R_f(\frac{1}{\tilde{\kappa}_3 n}). \tag{7}$$

Then $c_1(n) < c_2(n)$ follows from Proposition 4. In addition, by the definition of $c_1(n)$ and $c_2(n)$, when $c_a - c_f \in (c_1(n), c_2(n))$, for $\pi = 0$ the market is more profitable, but for $\pi = \frac{1}{\tilde{\kappa}_3 n}$ the auction is more profitable. Direct substitution into the bounds developed in the proof of Proposition 4 shows that as n grows, $c_1(n)$ converges to zero while $c_2(n)$ remains bounded away from zero. Denote $c_1 = \limsup c_1(n)$ and $c_2 = \liminf c_2(n)$. For n large enough the statement of the corollary thus follows for $\pi = 1/\tilde{\kappa}_3 n$, but then by continuity it also follows for a range of π in its neighborhood (given n).

Proof of Corollary 6. We first consider $\pi = 0$. In this case, by the proof of Corollary 5, for n large enough the seller chooses the fixed-price market. We show that, in that market and for n large, production is ε-efficient. In that market, for n large the seller sets a price in $[v_M, v_H]$, and using the proof of Proposition 4, we can bound $R_f(0) = p^*(1 - F(p^*)^n) \geq p^*(1 - \frac{1}{np^*\underline{f}}) = p^* - \frac{1}{n\underline{f}}$. Moreover, from the first-order condition (5) we have $F(p^*)^{n-1} \geq 1/(1 + nv_H\bar{f})$ and therefore for n large $-(1 - F(p^*)) \geq \log(F(p^*)) \geq -\frac{\log(n)}{n-1} - \frac{\log(2v_H\bar{f})}{n-1}$. Since $1 - F(p^*) = \int_{p^*}^{v_H} f(t)dt \geq \underline{f}(v_H - p^*)$, it follows that for n large $v_H - p^*$ is bounded from above by a constant times $\log(n)/n$. Thus $v_H - R_f(0)$ goes to zero as n grows, and for any $\varepsilon > 0$ production becomes ε-efficient for n large enough.

Now suppose that $\pi = \frac{1}{\tilde{\kappa}_3 n}$. In the fixed-price market the seller still sets a price below v_H, so revenue is between v_H and $R_f(0)$. Hence $v_H - R_f(\pi)$ also goes to zero at a rate of $\log(n)/n$, and production is ε-efficient for n large. In addition, we know from Corollary 5 that the seller prefers the auction and that the revenue gap between the auction and the fixed price market, $R_a(\pi) - R_f(\pi) - (c_a - c_f) = c_2(n) - (c_a - c_f)$, is positive and bounded away from zero as n grows. Let $\tilde{\kappa}_4$ denote a positive lower bound for this term. Since production is ε-efficient in the market, and the auction offers a premium in net revenue of more than $\tilde{\kappa}_4$, it follows that when $2\varepsilon < \tilde{\kappa}_4$ for n large there is ε-overproduction in the auction. This argument establishes the desired result for $\pi = \frac{1}{\tilde{\kappa}_3 n}$, but continuity, and the fact that we define both ε-efficiency and ε-overproduction with strict inequalities, implies that the result also holds for a range of π in its neighborhood.

Proof of Corollary 7. Consider first the gain from one more buyer in the fixed-price setting with a share π of fools. To establish the result for integer n, it is useful to first consider the optimization problem (1) for any positive real n and treat n as a continuous variable. Then, the increase in revenue from a marginal increase in n can be computed using the envelope theorem, and equals

$$-p(1 - \pi)^n F(p)^n(\log(1 - \pi) + \log F(p)).$$

Now fix n_0 at an integer value, and let $\pi = 1/(\tilde{\kappa}_3(n_0 + 1))$. Then as long as $n \in [n_0, n_0 + 1]$ the optimal price is interior. Consider a marginal increase in n when it is in this interval. For any constant $k > 0$, if n_0 is sufficiently large the above expression will be bounded in absolute value by k/n_0. This is because $(1 - \pi)^n F(p)^n$ is by (6) of order $1/n$ and – since p converges to v_H and $\pi = 1/(\tilde{\kappa}_3(n_0 + 1))$ to zero – $\log(1 - \pi) + \log F(p)$ becomes arbitrarily close to zero as n_0 grows. Integrating across the marginal increases as n runs from n_0 to $n_0 + 1$, it follows that for any k there is an n_0 large enough such that the impact of one more buyer on revenue is less that k/n_0. The same argument also applies when we instead set $\pi = 0$, and implies statement (1).

Next consider the gain from one more buyer in the auction. Here the gain is at least

$$[q_2^{n_0+1}(\pi) - q_2^{n_0}(\pi)](v_B - v_H) = (1 - \pi)^{n_0 - 1}\pi^2 n_0(v_B - v_H),$$

which, for $\pi = 1/(\tilde{\kappa}_3(n_0 + 1))$ is bounded from below by some constant times $1/n_0$. It follows that the gain from an additional buyer in the auction exceeds that in the market when n_0 is large, so we can find a small enough k_b that ensures recruitment is optimal in the auction but not in the fixed-price market. Finally, for n_0 large the seller will prefer the auction to the market if there are

n_0 buyers and $\pi = 1/(\tilde{\kappa}_3(n_0+1))$, by a margin that does not vanish as n_0 grows. This follows from the bounds developed in Proposition 4 through the same logic as in the proof of Corollary 6.

Appendix B: Appendix-Tables

Table B.1: Auction House Categories

Christie's	
Ancient Art & Antiquities	Books, Manuscripts & Maps
Asian Art	Fine Art
Collectibles	Jewelry & Watches
Furniture & Decorative Art	Wine, Spirits & Cigars
Photographs, Prints & Multiples	
Sotheby's	
Ancient and Ethnographic Arts	Musical Instruments
Asian Art	Paintings, Drawings and Sculpture
Books and Manuscripts	Photographs
Ceramics and Glass	Prints
Collectibles and Memorabilia	Silver, Russian and Vertu
Fashion	Stamps, Coins and Medals
Furniture and Decorative Arts	Watches and Clocks
Jewelry	Wine
Stockholm Auction House	
Arms, Armor, Guns & Firearms	Rare Book & Manuscripts
Clocks & Watches	Russian Auction
City Auctions	Selected Wine & Spirits
Fine Art & Antiques	Swedish Art Glasses
Modern Art & Works of Art	Toys, Technica & Nautica
Prints	
Lyon and Turnbull	
Lyon and Turnbull	
Asian Works of Art	Jewellery & Watches
Books, Manuscripts & Photographs	Paintings & Prints
Ceramics & Glass	Rugs & Carpets
Decorative Arts & Design	Silver & Objects of Vertu
Furniture, Works of Art & Clocks	Sporting & Arms and Armour

Bonham's

Angling & Fishing	Made In California
Antiquities	Modern and Contemporary Art
Antique Arms & Armor	Musical Instruments
Sporting Guns	Native American & Pre-Columbian Art
California and American Paintings	Natural History
Asian Art	Rugs & Carpets
Blue Printed Earthenwares	Pianos
Books, Maps & Manuscripts	Photography
Cameras	Portrait Miniatures
Design 1860 - 1945	Prints
Clocks & barometers	Railwayana
Coins & Medals - Glendining's	Rivercraft & Maritime
Contemporary Asian Art	Russian
Contemporary Ceramics	Scientific Instruments
Entertainment Memorabilia	Sculpture & Works Of Art
Islamic & Indian Art	Silver
Contemp. Indian & Pakistani Paintings	Single Owner Sales
Costume & Textiles	Sports Memorabilia
Design 1860 - 1945	Staffordshire Figures
European Pictures	Stamps
European Porcelain, Pottery & Glass	Soma Estate Auctions
20th Century British Art	Sunset Estate Auctions
Furniture	The Dog Sale
20th Century Decorative Art	Toys & Dolls
Jewelry	Urban Art
Mechanical Music	Watches & Wristwatches
Motoring	Wine
Contemporary Art	

Note: List of item categories from the major auction houses: Christie's, Sotheby's, Bonhams 1793, Stockholm Auction House, and Lyon and Turnbull as of 6/22/2008.
Sources:
www.christies.com ["Categories"];
www.sothebys.com/app/live/dept/DeptTopicAreaMainLive.jsp;
www.bonhams.com/cgi-bin/public.sh/pubweb/publicSite.r?sContinent=EUR&screen=menuDepartments ["USA"];
www.auktionsverket.se/ramsidor_08/engelsk_ram.html ["Auctions"];
www.lyonandturnbull.com/departments.asp.